

AN INVESTIGATION OF THE STABILITY OF A
SYSTEM OF TWO SHIPS EMPLOYING AUTOMATIC
CONTROL WHILE ON PARALLEL COURSES IN
CLOSE PROXIMITY

Charles Natale Calvano

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by

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ABSTRACT

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Submitted to the Department of Naval Architecture and Marine Engineering on 21 May 1970 in partial fulfillment of the requirements for the degrees of Master of Science in Ocean Engineering and Naval Engineer.

An introduction outlining the development of ship operations in close proximity for underway replenishment is followed by a survey of the literature dealing with the measurement of interaction effects between ships under such circumstances. A brief outline of the theoretical treatment of ship motions in open water is given and followed by the modifications to the theory required for treatment of the two ship system. The applicable equations are written for calm water and suitably linearized. The techniques of treating the differential equations as algebraic equations, and of solving the equations by determinant evaluation are described.

Extension of available interaction and open water data for two different ships to a full data set for a single, modified ship is carried out, and the numerical values of resulting hydrodynamic derivatives are given. The numerical solution of the equations and the extraction of stability roots are then described and results for 91 typical trial data sets are listed. A discussion of the results obtained is followed by an evaluation of the correctness of the solution method employed.

The computer programs used throughout are described in detail in four appendices.

Thesis Supervisor: Martin A. Abkowitz
Title: Professor of Naval Architecture

ACKNOWLEDGEMENT

Thesis acknowledgements traditionally single out the thesis supervisor as the primary source of aid to the author. This tradition is doubly applicable to this thesis, for reasons other than the usual ones. First, Professor Martin A. Abkowitz is largely responsible for previous work in the field of ship motion which made possible the mathematical approach used herein, and one need only refer to Chapter II to determine the extent of the author's reliance on his efforts. Second, he greatly enhanced the effectiveness of this thesis effort in fulfilling its function as a learning process, by his adherence to the principle that the thesis should "comprise an original investigation.....by a single student", (as it is supposed to), and by refraining from constant "second guessing" of the author's work.

As a result, of course, the errors inevitably included herein, remain the "property" of the author alone.

My wife, Charlotte, through her hours of listening to discussion of a subject with a limited range of interest, contributed significantly to the timely completion of the project.

Mr. David Burke, a fellow graduate student, who on at least two occasions helped me to see the forest when the trees had obscured my view, also deserves the author's thanks.

And Miss Sandra Vivolo, secretary in the Department of Naval Architecture and Marine Engineering, patiently endured the never-ending changes and queries of "is that chapter finished yet?", that any typist knows so well.

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Chapter I

INTRODUCTION

I. Background-Replenishment at Sea

In their efforts to achieve a degree of independence from land bases, the navies of the world have, for decades, practiced various methods of underway refueling and replenishment of their ships. As techniques were improved and experience was gained, these operations progressed from the status of infrequent, makeshift affairs, to that of routine evolutions. These replenishment methods have ranged from actually lashing together the supply ship and receiving ship while passing coal and provisions from one to the other, (one assumes this was attempted in only the very calmest of seas, or in sheltered roadsteads), to the transfer of cargo via "vertical replenishment" (VERTREP) helicopters. Other methods employed over the years have included the astern refueling method and the alongside method for transferring fuel and cargo.

In the former, the receiving ship recovers a fuel hose trailed in the water by the tanker and takes on fuel while steaming in the tanker's wake. In the alongside method, the receiving ship overtakes the supply ship on a parallel course, laterally separated by from 40 to 180 feet. The transfer is effected while the ships remain abreast at this distance.

The alongside method is the standard of the U.S. Navy, (augmented by VERTREP for cargo transfer, though not, of course, for refueling), and such replenishment operations

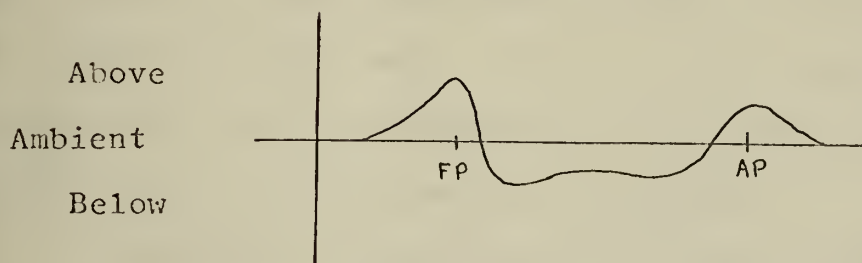
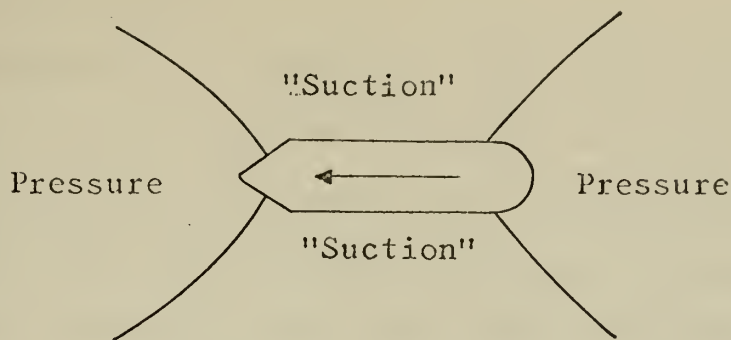
have become almost a way of life for U.S. combatant ships. Unless, and until, all U.S. combatant ships become nuclear fueled, this method will probably remain in wide use.

Of course there are other situations than replenishment when two ships might wish to operate in close alongside proximity while underway. To generalize the treatment herein, the forces and moments arising from the tension in the replenishment rig lines is not included. Thus the situation occurring whenever two ships must steam alongside each other at close quarters is that which is treated in this thesis.

II. Some Problems

Although alongside replenishment has become commonplace, this type of close-in steaming has not ceased to be a dangerous undertaking. There are, of course, dangers arising from the lack of freedom to maneuver to avoid other shipping, or, in time of war, the actions of an enemy. But the ever-present source of danger to two ships steaming close together on parallel courses is this very proximity of the ships, causing each to constitute a collision hazard for the other.

While alongside, the ships must make almost constant rudder and engine speed adjustments to maintain position. Because the ships are close, the presence of each affects the forces and moments on the other through the influence of each on the pressure field and flow patterns around the other. (See Figure I-1). Thus, neither ship will respond as it does in open water where these influences are not present. And



Pressure Field about a Ship in Open Water

(From Reference A-1)

FIGURE I-1

as the ships alter their relative positions, the direction and magnitude of these influences are continuously altered.

The repeated corrections required over a period of up to several hours, introduce hundreds of opportunities for human error or momentary lack of attention to give rise to damaging consequences in the form of a collision. Any method or device which could serve to reduce the probability of such errors occurring would serve to lessen a primary hazard for two ships operating in this way.

III. Automatic Control

The introduction of automatic rudder angle and engine speed control to the alongside operation is an attractive prospect. Such a control system would employ sensors to measure longitudinal and lateral separation of reference points on the two ships, (and their rates of change), and compare these values to desired equilibrium values. It would then generate rudder and engine orders to reduce the deviations to acceptable magnitudes. Such a system could be a boon to conning officers and helmsmen by relieving them of much of the tedium of maintaining position alongside.

A theoretical investigation of such a system is the subject of this thesis. The problem will be approached from a mathematical standpoint as a two-body system of ship motions. The hardware required to make it operable will not be examined or described. It will be assumed that a sensor, (doppler sonar, for instance), is available to continually measure lateral and longitudinal ship separations and their rates of change.

Once the deviation from desired ship separation and heading is known, an automatic control system can order rudder and engine speed changes to reduce the deviations. The various constants associated with a theoretical control system meeting this description will be a part of the mathematical model of the two ship system. The stability of the system will be investigated, to determine if it appears feasible to employ such an automatic control system to keep the ships in position alongside. (By stability is meant the tendency of the two

ship system to return to an equilibrium condition after receiving a small disturbance from that equilibrium condition. If the system "returns, or tends to return, to the equilibrium condition when the disturbance is removed, then it is stable". [A-3]*)

IV. Historical Aspects---The Literature

A. Taylor.

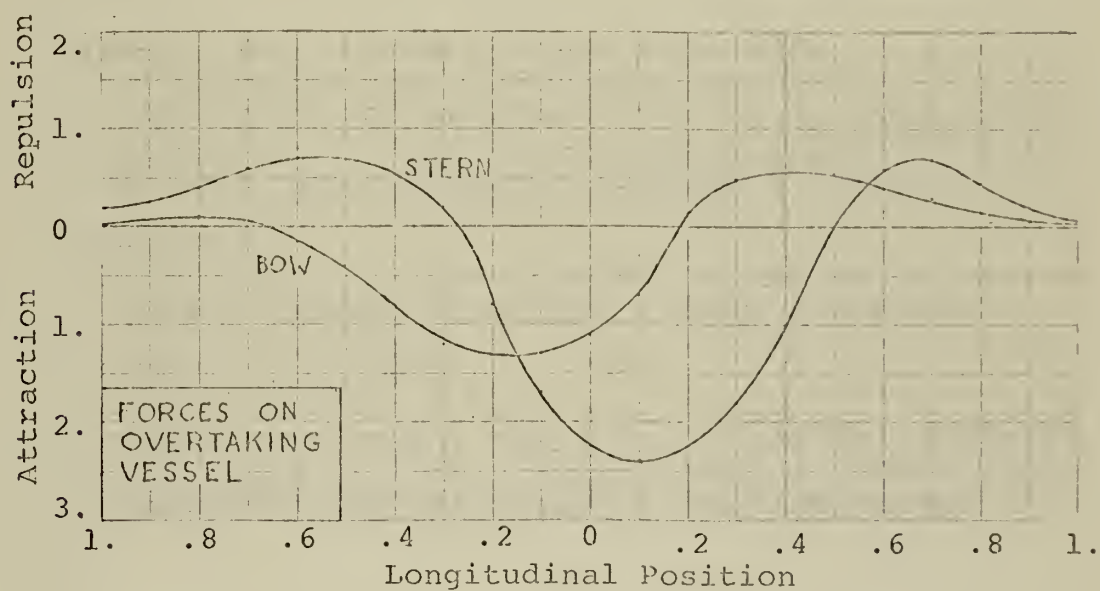
As far as the author has been able to determine, the first systematic attempt to measure interaction phenomena for two ships on parallel courses in open water was made by David W. Taylor, the ubiquitous Naval Constructor. He conducted a series of tests utilizing four models, each of 3,000 pounds displacement, at the Model Basin in Washington, in 1909. [A-2]

Taylor's tests were conducted by towing the models in pairs, either abreast of one another, or with one model a fixed distance ahead of the other. The models were not self-propelled, and did not have propellers affixed. They were towed under towing carriages, with their bows and sterns prevented from translating laterally, but free to move vertically, changing draft and trim while being towed. Water depths were "many times the draught of the models". Attraction and repulsion forces were measured at the attachment points near the bow and stern of each model. Table I-1 lists the characteristics of the models employed and Figure I-2 is a typical plot of the results Taylor obtained.

*Letter-number citations in brackets refer to the references listed at the end of the body of this thesis.

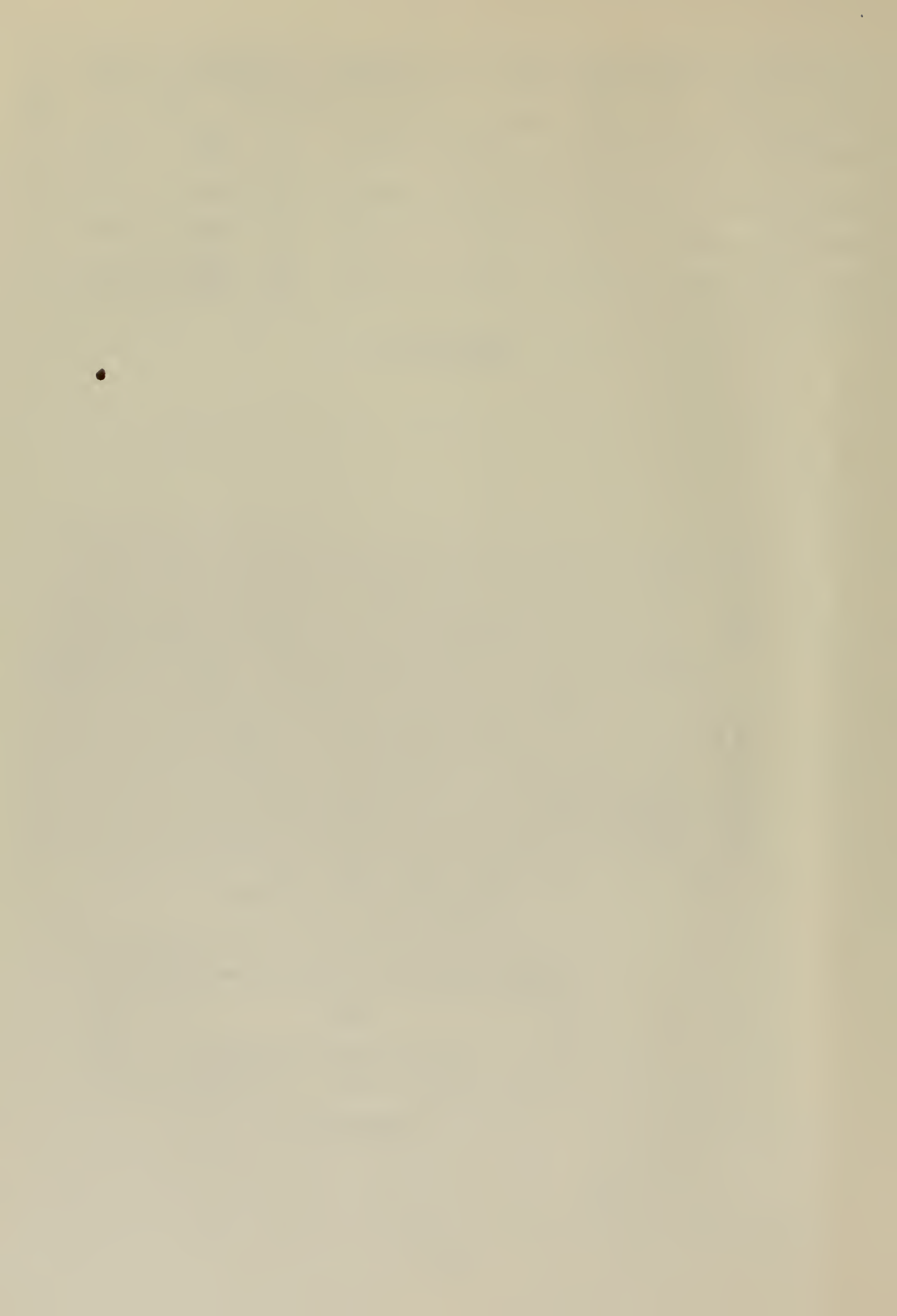
| Model # | Beam (B) ft. | Draft (H) ft. | $\frac{B}{H}$ | Prismatic Coeff C_p | Midships Coeff C_m | Block Coeff C_b |
|---------|-----------------|------------------|---------------|--------------------------|-------------------------|----------------------|
| 834 | 3.692 | 1.263 | .560 | .560 | .900 | .504 |
| 838 | 3.503 | 1.198 | 2.92 | .560 | 1.00 | .560 |
| 858 | 3.586 | .957 | 3.75 | .740 | .926 | .685 |
| 866 | 2.778 | 1.235 | 2.25 | .740 | .926 | .685 |

TABLE I-1



Side Force in Units of Resistance
vs.
Longitudinal Position

Figure I-2



In conducting his tests, Taylor found that he had difficulty towing the models on a straight course. In view of the fact that towing carriages were used, this is hard to understand, but was explained no further. As a result of this difficulty, he felt that the forces measured could not be regarded as highly accurate, but felt that they showed the tendencies and general nature of the interaction, or, to use his term, "suction", phenomena.

The sequence of events, when vessel B overtakes vessel A on a parallel course, seemed to be about as follows:

1. As B's bow overlaps A's stern, a small repulsion force is felt at both B's bow and stern.
2. As B continues to overtake A, B's bow is attracted, while its stern is repelled, causing B to tend to turn into A.
3. As B draws abreast of A, both B's bow and stern are drawn in toward A.
4. As B pulls up farther, the force on its bow switches to repulsion, while its stern is still attracted to A.
5. As B draws farther forward, to where its stern is approximately abreast of A's midlength, both B's bow and stern are repelled.

Taylor noted further, that the occurrence of these events was somewhat more consistent with the "finer" models, while the results with the "full" models tended to be erratic. The effect of the "suction" forces on the models' resistance was

not measured, but Taylor did note that the side forces "varied with speed as the resistance of the model"; that is the interaction forces increased as the square of the velocity.

It is evident from the discussions accompanying Taylor's paper that there was disagreement within the Naval Architect community concerning the significance and, in fact, even the existence, of interaction forces for full scale ships.

B. Gibson and Thompson.

In 1913, Gibson and Thompson conducted a series of full scale tests with small steam launches in the Thames River near London, to verify the existence, and measure the effect of interaction phenomena. [B-2] They did note effects similar in type to those described by Taylor, but due to instrumentation difficulties, their report contains no quantitative information of a useful nature in considering interaction effects for ships.

C. Robb.

A disastrous collision between the liner QUEEN MARY and the destroyer H.M.S. CURACOA in 1942 led to a series of experiments reported by A.M. Robb in 1949. [A-8] In these tests, the models used were of these two ships and made to a 1:56 scale. This resulted in the QUEEN MARY model being "rather more than 18 feet" in length, and that of the CURACOA "rather more than 8 feet". In these experiments, both models were self-propelled, with first one, then the other, being constrained to run on a straight course while the other passed

on a nearly parallel course. Robb again confirmed the existence of significant interaction effects drawing the smaller ship into the larger, in some circumstances, and his article is accompanied by several sketches showing the trajectories of the ships for various speeds and lateral separations. The results presented are largely qualitative in nature, and do not disagree with those obtained by Taylor 40 years earlier.

But again, in the discussions of Robb's paper, the fact that the very existence of interaction effects was doubted in some quarters was brought out. [B-5] Professor Prohaska, in his discussion, stated, "The question of interaction between passing vessels has always given rise to interesting exchanges of opinion.....a great many experts, amongst whom are numerous seafaring people with great experience, deny the existence of interaction, or at least think it without importance in unrestricted water." (Prohaska, however, did not share this view.)

D. Restricted Water Studies.

Over the years, there have been numerous other experiments and theoretical developments concerning the interaction between a ship and a stationary, constant-geometry object, such as a canal wall or bank. References A-6, A-10, B-6, B-10, B-11, B-12, B-13 and B-18 all deal with this phenomenon. A ship, being a large object, can appear, in its effects, as a wall would appear to another ship passing near. But of course, the geometry of the ship-object is different than when the object is a wall. And, also, neither ship is immovable, as is the wall.

Most of the investigations concerning interaction in canals or channels include shallow water conditions. The shallow water tends to emphasize the interaction effects caused by the wall. This investigation, though, is concerned with the interaction between ships in deep water.

E. Newton.

The single study of interaction effects which proved to be of greatest use to the author in attempting to obtain quantitative information concerning the interaction forces and moments was that listed as reference A-1, by R.N. Newton and reported at the First Symposium on Ship Maneuverability conducted at the David Taylor Model Basin in 1960.

Newton reported on experiments conducted at the Admiralty Experiment Works (AEW) in England, between 1946 and 1948 and which were inspired by the growth of alongside replenishment experienced during the war. In his paper, Newton acknowledges the pioneering work of Taylor in this field and laments the near-total lack of work in this area in the years between Taylor's tests and the AEW efforts, although he noted the wealth of information available to him concerning interaction effects in restricted channels and shallow water.

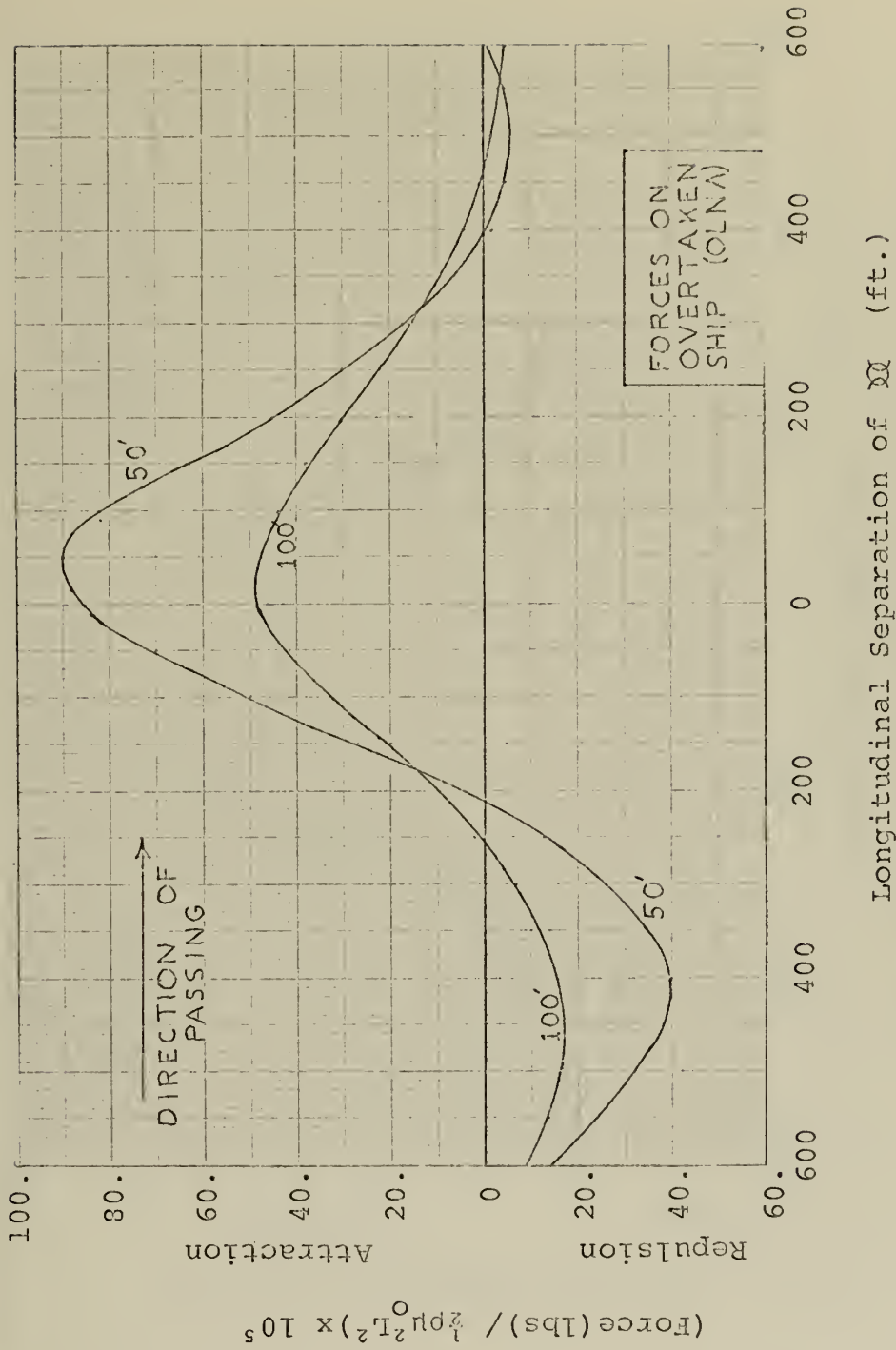
Newton's experiments were with models of a battleship, H.M.S. KING GEORGE V and a replenishment supply ship, R.F.A. OLN.A. (See Table I-2 for ship characteristics.) The tendencies exhibited by the models were confirmed by full scale trials at sea.

| | K.G.V | OLNA |
|---------------------------|------------|--------|
| Length on W.L., ft. | 740 | 547 |
| Beam, ft. | 103 | 70 |
| Draft, ft. | 29.3 | 30 |
| Displacement, tons | 39,890 | 23,570 |
| Block Coefficient | .611 | .714 |
| Corresponding water depth | 75 fathoms | |

TABLE I-2

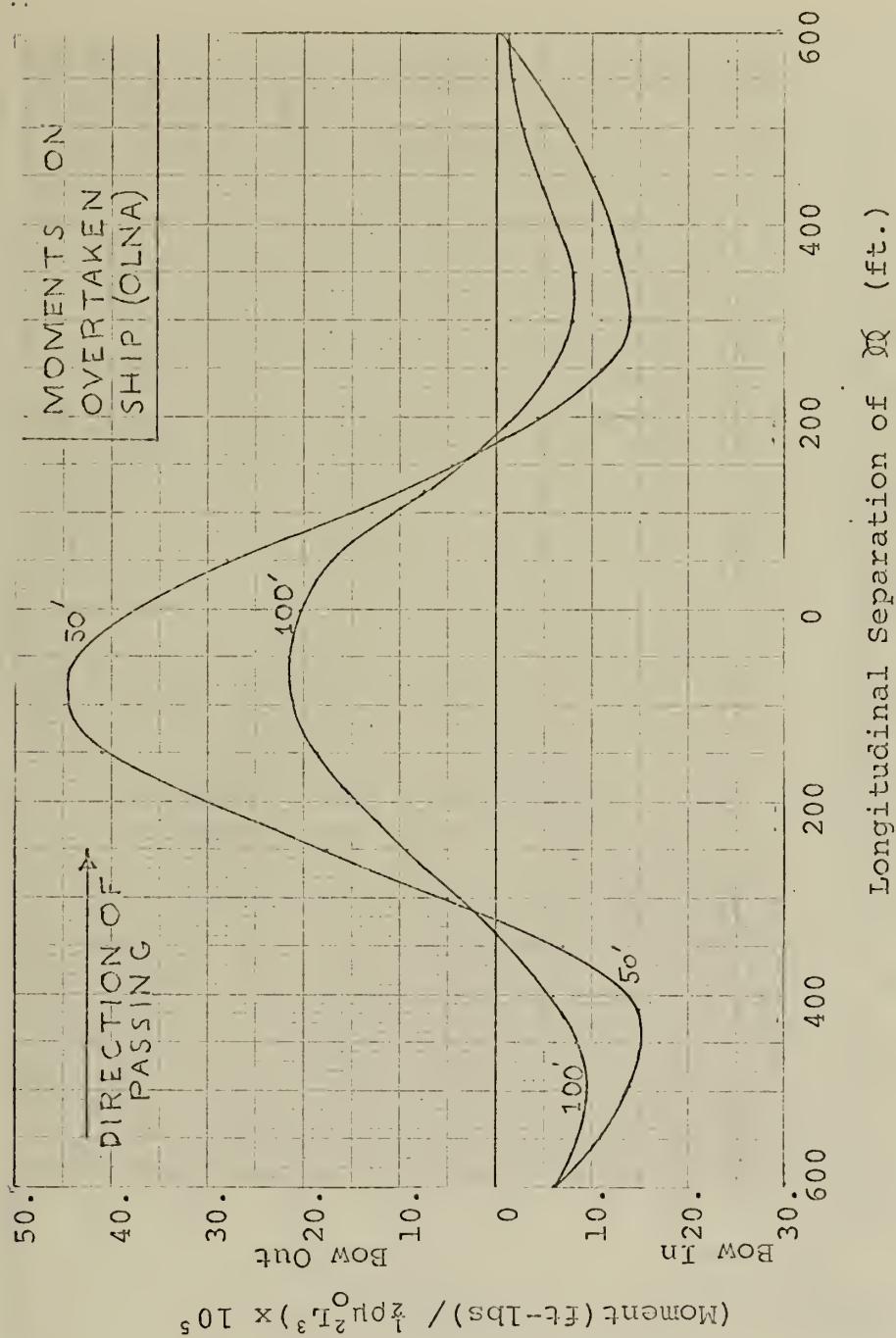
Tests were conducted with both towed and self-propelled versions of the models. The models were allowed complete vertical freedom, but constrained from lateral motion. Forces and moments were measured at longitudinal separations of the midships sections of the ships from 600 ft. aft of the abreast position to 600 ft. forward of this position, at two lateral separations; 50 and 100 ft. side to side distance. With the ships in the abreast position, the side to side distance was varied from 25 to 140 ft. (All distances are the corresponding full scale values.)

Figures I-3 and I-4 show the longitudinal variation of the forces and moments experienced by OLNA at the two lateral separations and Figure I-5 shows the lateral variation with the ships abreast. These data formed the basis for quantitative interaction forces and moments used in this investigation, as OLNA is not radically different in hull characteristics from the MARINER class vessel used for this study. In the figures, OLNA is the ship being overtaken. The agreement with the results obtained by Taylor is apparent.



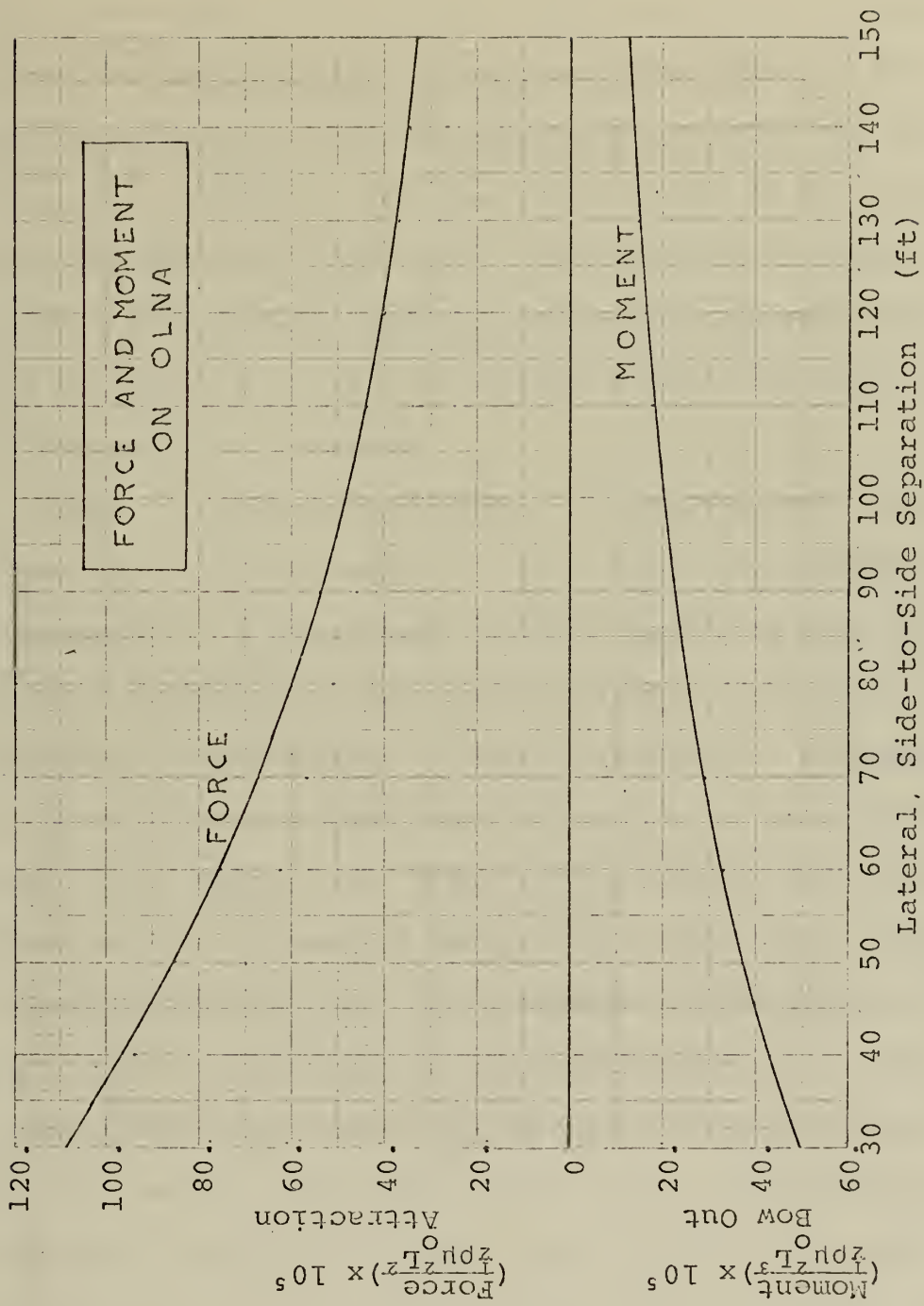
Non-Dimensional Force vs. Longitudinal Separation
 for Side-to-Side Separations of
 50 and 100 feet

Figure I-3



Non-Dimensional Moment vs. Longitudinal Separation
for Side-to-Side Separations of
50 and 100 feet

Figure I-4



Non-Dimensional Force and Moment
vs.
Lateral Separation
at 0-ft. Longitudinal Separation

Figure I-5

F. Strom-Tejsen and Chislett.

In making a theoretical investigation such as this one, it is necessary to have realistic values for the hydrodynamic derivatives employed in the mathematical model. (The use of these derivatives, and the mathematical model are discussed in Chapter II, THEORY.) For two ships steaming in proximity, values of both the open water and interaction derivatives are needed, as forces and moments which are always present for a ship in open water will be present along with those caused by the interaction phenomena.

Thus, in this investigation it was necessary to find sources for typical values of both kinds of hydrodynamic derivatives. As mentioned earlier, Newton's work with KING GEORGE V and OLN A was the only available source of quantitative interaction information. It was, therefore, necessary to find values of the open water derivatives for a ship similar to either KING GEORGE V or OLN A. Strom-Tejsen and Chislett in reference A-5 determined most of the open water hydrodynamic derivatives of interest for a MARINER class merchant ship. (See Table I-3 for MARINER characteristics.) In reference A-4, Strom-Tejsen used these derivatives to theoretically predict the open water maneuvering behavior of a MARINER, and subsequent sea trials proved these values to be quite accurate. Fortunately, OLN A and the MARINER class hulls are sufficiently similar to expect the hydrodynamic derivatives of one to reasonably resemble those of the other. Therefore, in the development of the results of this thesis, the open water

derivatives of the MARINER were used, and, to account for interaction effects, derivatives derived from the interaction force and moment data of the OLNA experiment were added to these to obtain the entire list of needed derivatives. Thus, the ship type investigated is a somewhat modified MARINER class ship.

| MARINER Characteristics | |
|-------------------------------|--------|
| Length, ft. | 528.5 |
| Beam, ft. | 76.0 |
| Draft, ft. | 29.75 |
| Displacement, tons | 20,900 |
| Block Coefficient C_b | .6125 |
| Prismatic Coeff. C_p | .6246 |
| Midships Section Coeff. C_m | .9807 |

TABLE I-3

Chapter II

THEORY

I. Background

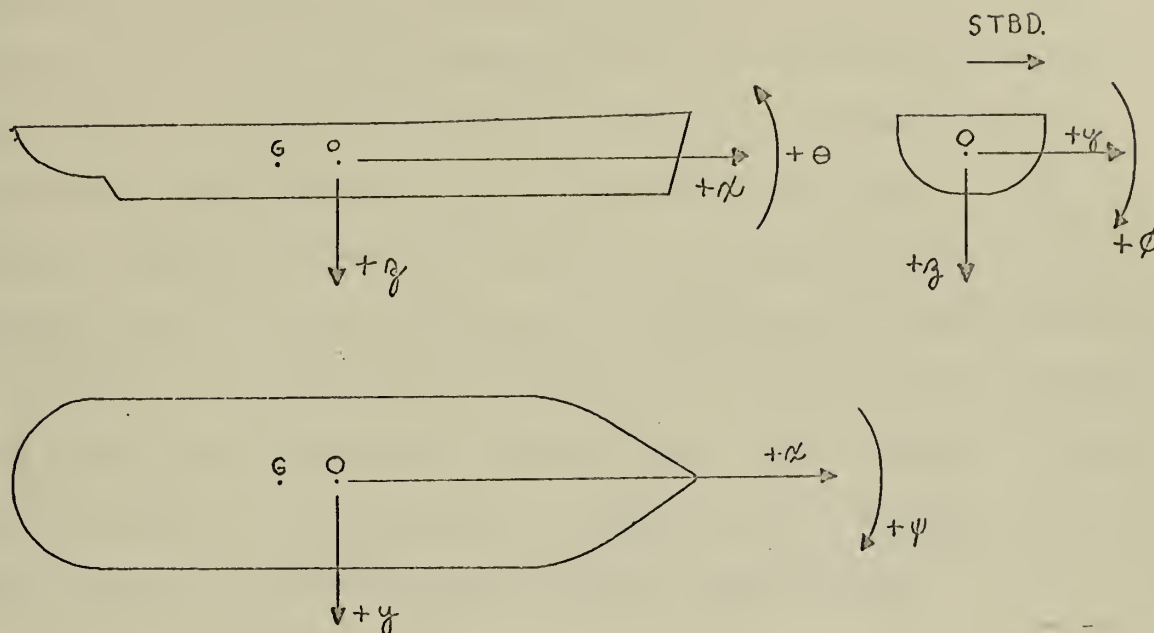
The derivation of a mathematical model representing the steering and maneuvering of a surface ship in open water is given by Abkowitz in reference A-3. In its general form, the model accounts for non-linear, as well as linear, effects, and in his computerized prediction of open water, surface ship maneuvers, Strom-Tejsen uses the Abkowitz model with up to third order terms. [A-4]

This mathematical model, in linearized form, serves as the basis for the analysis made in this thesis, with appropriate extension of the model to include the interaction effects which are not present in open water, but which are important for two ships on parallel courses in close proximity. The model used herein also is modified to represent the two-ship system, rather than the motions of a single ship, and accounts for the fact that both ships are free to move in the horizontal plane. (For a more complete description than the outline which follows, of the derivation of the open water model, reference A-3 is suggested).

II. Reference Axes

The reference axes used constitute an orthogonal, right-handed system which takes advantage of the existence of a vertical plane of symmetry through the centerline of the ship. The selection of the axis system illustrated introduces

simplifications in the equations of motion by virtue of the fact that the plane of symmetry of each ship is parallel to the principal axes of inertia of the ship.



(See Table II-1 for definitions of all symbols used in this chapter)

Reference Axis System

FIGURE II-1

III. Equations of Motion for a Ship Moving in the Horizontal Plane

A general form of the equations of motion for a body which is allowed to move in all the six degrees of freedom is given by Abkowitz.

Normally, when dealing with steering and maneuvering of surface ships in open water, the primary motions are considered to take place in the horizontal plane, and vertical motions are

neglected. As this is also the case for the two-body system of interest, where only horizontal motions are of concern, the equation in Z, the vertical force equation, can be dropped. Further, choice of the axis system parallel to the plane of symmetry of the ships eliminates the cross terms of inertia, and, if it is assumed that the center of gravity of each ship is in its centerline plane, (as, indeed, can be expected), the quantity y_g is zero for both ships. The usual assumption that rolling and pitching motions, if not extreme, have little influence on steering and maneuvering is also made, allowing the equations for K and M to be neglected, and making the variables p and q zero. (See Table II-1 for definitions). The equations of motion for a single surface ship moving in the horizontal plane thus become:

$$\begin{aligned} \text{a) } X &= m[\dot{u} - rv - x_g \dot{r}^2] \\ \text{b) } Y &= m[\dot{v} + ru + x_g \dot{r}] \\ \text{c) } N &= I_z \dot{r} + mx_g (\dot{v} + ru) \end{aligned} \quad (2-1)$$

(where the notation $\dot{v} = \partial v / \partial t$, is used)

The left-hand sides of these equations represent the sum of all forces in the x and y directions and all moments about the z axis, respectively. The right-hand sides are the dynamic response terms associated with the ship and its motions.

IV. Taylor Expansion of Forces and Moments

The forces and moments on the left-hand sides of the equations of motion can be expressed as functions of the properties of the body, properties of the fluid and properties

TABLE II-1

| Definitions of Symbols | |
|--------------------------|---|
| x_o, y_o, z_o | Reference coordinates fixed with respect to earth. |
| x, y, z | Reference coordinates fixed in ship. |
| u, v, w | Velocities in direction of x , y and z axes, respectively. |
| ϕ, θ, ψ | Roll, pitch and yaw angles about x , y and z axes, respectively. |
| p, q, r | Angular velocities about x , y and z axes, respectively. ($p=\dot{\phi}$, $q=\dot{\theta}$, $r=\dot{\psi}$). |
| δ | Rudder deflection in radians (port rudder is positive). |
| n | Propeller speed in revolutions per second. |
| X, Y, Z | Sums of forces acting on ship in x , y , z directions, respectively. |
| K, M, N | Sums of moments acting on ship about x , y , z axes, respectively. |
| α | Longitudinal separation of ships measured from Ship A midships section to Ship B midships section. |
| β | Lateral separation of ships measured from Ship A centerline to Ship B centerline. |
| Y_{u_B} | Typical of notation used for hydrodynamic derivatives. In this case: $\partial Y / \partial u$ for Ship B, measured when all variables but u are at equilibrium values. |
| $\Delta t_1, \Delta t_2$ | Control system time lags associated with rudder and propeller response, respectively. (Subscripts A or B indicate applicable ship, as Δt_{1_A}). |

k_{i_A}, k_{i_B}

Control system constants, $i=1$ to 6 , for Ship A or B. Constants associated with control system response to deviations from equilibrium, as

$$\delta_A = k_{1_A} \Delta\psi_A.$$

x_g, y_g, z_g

Distance of center of gravity of ship, (G), from the axis origin, (O), in x, y, z directions respectively.

m_A, m_B

Mass of Ship A or B in units of $(\text{lb sec}^2/\text{ft})$

\mathcal{D}

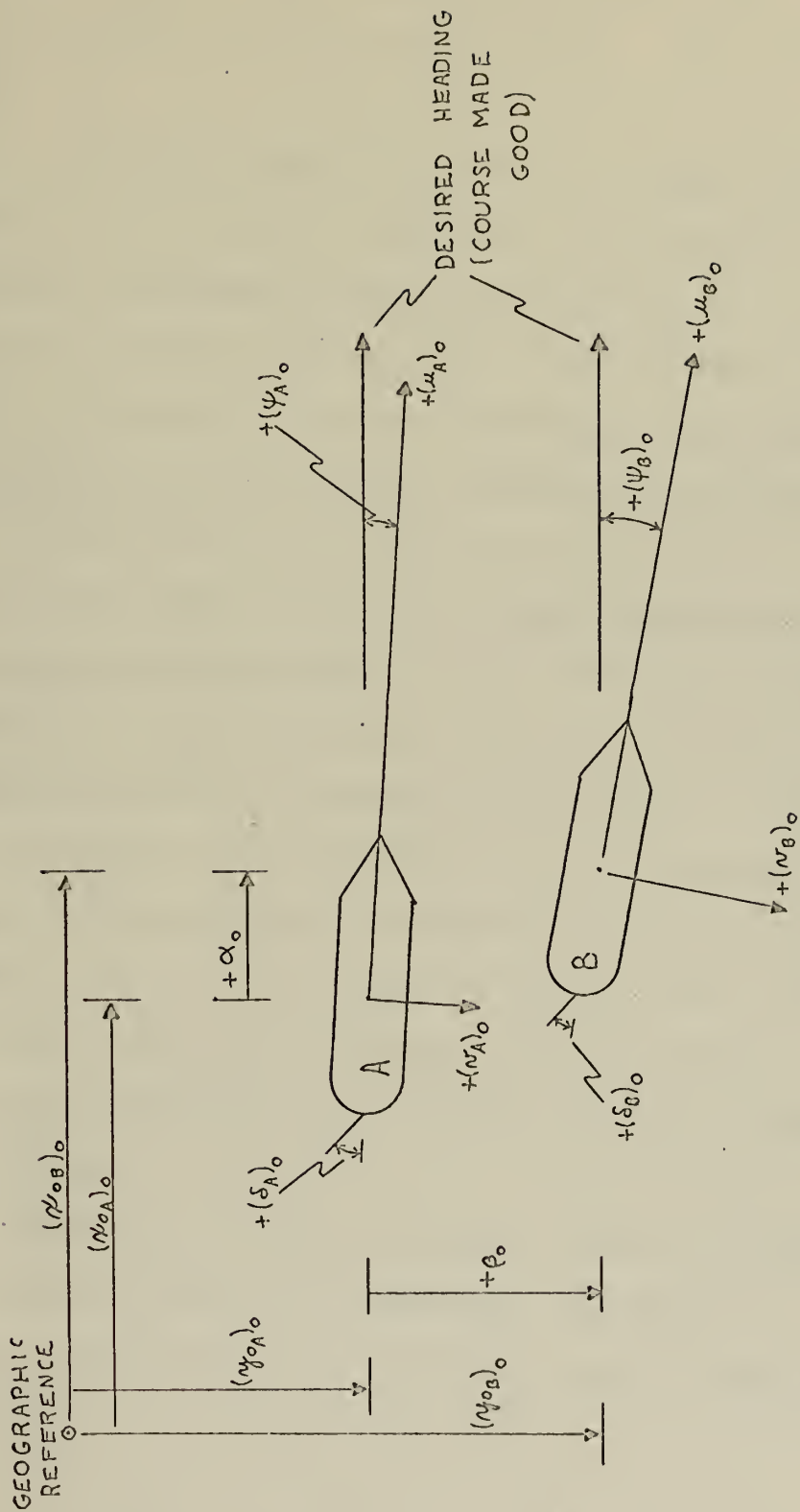
The operator $\partial/\partial t$, as $\mathcal{D}u = \partial u/\partial t$

of the motion. When considering specific hull forms, and using generally accepted scaling laws, the forces and moments may be considered functions of only the motion and orientation parameters of the bodies. (When dealing with automatic control of rudder angle and propeller speed, they are also functions of rudder deflection, δ , and propeller speed, n .)

The reference system for the two-ship system at equilibrium and an illustration of the variables involved are shown in Figure II-2.

When considering motion in an unrestricted horizontal plane, it is clear that no forces or moments arise from a change in ship location with respect to a fixed geographical reference point, allowing x_0 and y_0 to be discarded as variables of interest. Similarly, the equilibrium yaw angles, $((\psi_A)_0$ and $(\psi_B)_0$), or drift angles, can be dropped from the list of independent variables because forces and moments arising from a drift angle are accounted for by the terms in u and v . (If there were no drift angle, u would coincide with velocity made good in the desired direction, and v would be zero. Thus the forces and moments that $(\psi_A)_0$ and $(\psi_B)_0$ cause to be introduced are taken up in the terms in u and v . The inter-relation of drift angle and the independent variables, u and v , can be seen from $\tan(\text{drift angle}) = v/u$).

It is also possible to drop $\dot{\alpha}$, $\ddot{\alpha}$, $\dot{\beta}$ and $\ddot{\beta}$ as independent variables because of the simple trigonometric relations between them and v , \dot{v} , u and \dot{u} . The usual assumption that forces and moments produced by $\dot{\delta}$, $\ddot{\delta}$ and \dot{n} are negligible, in



Reference System for Two Ship System

FIGURE II-2

terms of ship control, is also made. (These forces and moments are not necessarily negligible when considering forces on the rudder stock, propeller blades, etc. But they do have a negligible effect on ship motion. (See reference A-4, pg. 7).)

As Abkowitz has shown, the left-hand sides of the equations of motion, comprising the functions describing the forces and moments acting on the ship, can be developed into useful form for analysis by use of the Taylor Series expansion for a function of several variables. The force or moment caused by a change in any variable is expressed as the product of the derivative of the force with respect to that variable, (with all other variables at equilibrium values), and the change in the variable. In the linearized form of the equations, only the linear terms are retained, though the expressions can be written to any desired degree of accuracy, by retaining sufficient terms in the expansion. (For example, the Y force caused by a change in u is written as $(\partial Y / \partial u) \times (\Delta u)$ where $(\partial Y / \partial u)$ is the value of $(\partial Y / \partial u)$ with all other variables at their equilibrium values. The derivatives, such as $(\partial Y / \partial u)$ are the hydrodynamic derivatives and can be experimentally determined.)

Employing standard notation, where Y_u represents $(\partial Y / \partial u)$ at equilibrium, the Y equations, for example, for the two ships, retaining only the linear terms of the Taylor expansion, become:

Ship A:

$$a) Y_{\alpha_A} \Delta \alpha + Y_{\beta_A} \Delta \beta + Y_{u_A} \Delta u_A + Y_{\dot{u}_A} \Delta \dot{u}_A + Y_{v_A} \Delta v_A + \dots$$

$$\dots + Y_{\delta_A} \Delta \delta_A + Y_{n_A} \Delta n_A = m_A [\dot{v}_A + r_A u_A + x_{g_A} \dot{r}_A]$$

$$b) Y_{\alpha_B} \Delta \alpha + Y_{\beta_B} \Delta \beta + Y_{u_B} \Delta u_B + Y_{\dot{u}_B} \Delta \dot{u}_B + Y_{v_B} \Delta v_B + \dots$$

$$\dots + Y_{\delta_B} \Delta \delta_B + Y_{n_B} \Delta n_B = m_B [\dot{v}_B + r_B u_B + x_{g_B} \dot{r}_B] \quad (2-2)$$

The left-hand sides of the X and N equations are similarly expanded for both ships.

V. Linearizing the Dynamic Response Terms

Since only linear terms have been retained in the expanded force and moment expressions on the left-hand sides of the equations, a similar linearization of the dynamic response terms on the right-hand sides must be made.

For all variables, the following holds true:

$$\text{variable} = (\text{variable})_o + \Delta \text{variable} \quad (2-3)$$

where the subscript "o" refers to the equilibrium value.

For all time derivatives of the variables, (\dot{v} , $\dot{\psi}$, etc.), the equilibrium value is zero, as the equilibrium condition is unchanging in time. Thus for these:

$$\text{variable} = (\text{variable})_o + \Delta \text{variable} = \Delta \text{variable} \quad (2-4)$$

$$(e.g. \dot{u} = (\dot{u})_o + \Delta \dot{u} = \Delta \dot{u})$$

Making use of (2-3) and (2-4), and by neglecting second

order terms consisting of products of small quantities, such as $(\Delta r) \cdot (\Delta u)$, the linearized right-hand sides of the Y equations become:

$$Y = m[\dot{v} + u_o r + x_g \dot{r}] \quad (2-5)$$

Similar developments are applicable to the X and N equations and are carried out in detail in [A-3].

VI. Linearized Equations of Motion

Taking into account all developments outlined thus far, the linearized equations of motion become:

Ship A:

$$a) \quad X_{\alpha_A} \Delta \alpha + X_{\beta_A} \Delta \beta + X_{u_A} \Delta u_A + X_{\dot{u}_A} \dot{u}_A + X_{v_A} \Delta v_A + X_{\dot{v}_A} \dot{v}_A$$

$$+ X_{\dot{\psi}_A} \dot{\psi}_A + X_{\ddot{\psi}_A} \ddot{\psi}_A + X_{\delta_A} \Delta \delta_A + X_{n_A} \Delta n_A = m_A \dot{u}_A$$

$$b) \quad Y_{\alpha_A} \Delta \alpha + Y_{\beta_A} \Delta \beta + Y_{u_A} \Delta u_A + Y_{\dot{u}_A} \dot{u}_A + Y_{v_A} \Delta v_A + Y_{\dot{v}_A} \dot{v}_A$$

$$+ Y_{\dot{\psi}_A} \dot{\psi}_A + Y_{\ddot{\psi}_A} \ddot{\psi}_A + Y_{\delta_A} \Delta \delta_A + Y_{n_A} \Delta n_A = m_A [\dot{v}_A + u_{o_A} \dot{\psi}_A + x_{g_A} \ddot{\psi}_A]$$

$$c) \quad N_{\alpha_A} \Delta \alpha + N_{\beta_A} \Delta \beta + N_{u_A} \Delta u_A + N_{\dot{u}_A} \dot{u}_A + N_{v_A} \Delta v_A + N_{\dot{v}_A} \dot{v}_A$$

$$+ N_{\dot{\psi}_A} \dot{\psi}_A + N_{\ddot{\psi}_A} \ddot{\psi}_A + N_{\delta_A} \Delta \delta_A + N_{n_A} \Delta n_A = I_{z_A} \ddot{\psi}_A + m_A x_{g_A} (\dot{v}_A + u_{o_A} \dot{\psi}_A)$$

d) $\left\{ \right.$
e) $\left\{ \right.$
f) $\left\{ \right.$

Corresponding Equations for Ship B

(2-6)

VII. Automatic Control

The automatic control system used to maintain each ship in position alongside is assumed capable of deflecting the rudder and changing propeller speed in response to measurement of the ship's heading angle error from equilibrium, $\Delta\psi$, and the separation parameter errors from equilibrium, $\Delta\alpha$ and $\Delta\beta$, as well as the rates of change of these variables. The rudder control is sensitive to ψ , $\dot{\psi}$, β and $\dot{\beta}$; the propeller control to α and $\dot{\alpha}$.

$$a) \delta = k_1 \Delta\psi + k_2 \dot{\psi} + k_3 \Delta\beta + k_4 \dot{\beta}$$

$$b) n = k_5 \Delta\alpha + k_6 \dot{\alpha} \quad (2-7)$$

(where k_i are the control system constants)

There will be time lags, (Δt) , in both control systems, representing the lapse between ordering a control response and attainment of the response. Thus rudder deflection at time t is proportional to ψ at time t_1 , (where $t = t_1 + \Delta t$).

Therefore:

$$\delta(t) = k_1 \psi(t - \Delta t) + k_2 \dot{\psi}(t - \Delta t) + k_3 \beta(t - \Delta t) + k_4 \dot{\beta}(t - \Delta t) \quad (2-8)$$

In the linearized equations of motion ψ and β are functions of t . Thus these variables in (2-8) must be obtained as functions of t , which is done through another Taylor expansion, carried out in detail in [A-3]. If the time lags are assumed small, only linear terms need be kept, and the automatic control system equations for δ and n become:

$$\begin{aligned} \text{a) } \delta = & [k_1 + (k_2 - k_1 \Delta t_1) \mathcal{D} - k_2 \Delta t_1 \mathcal{D}^2] \Delta \psi \\ & + [k_3 + (k_4 - k_3 \Delta t_1) \mathcal{D} - k_4 \Delta t_1 \mathcal{D}^2] \Delta \beta \end{aligned}$$

$$\text{b) } n = [k_5 + (k_6 - k_5 \Delta t_2) \mathcal{D} - k_6 \Delta t_2 \mathcal{D}^2] \Delta \alpha \quad (2-9)$$

(Where \mathcal{D} is the operator $\partial/\partial t$; e.g. $\mathcal{D}^2 \psi = \partial^2 \psi / \partial t^2$)

Here Δt_1 is the time lag associated with rudder response and Δt_2 that associated with propeller response.

These expressions for δ and n are substituted in the equations of motion.

VIII. Equations Relating α and β to u , v and ψ

Examination of Figure II-2 shows the following relationships to hold:

$$\text{a) } \dot{\alpha} = u_B \cos \psi_B - u_A \cos \psi_A + v_A \sin \psi_A - v_B \sin \psi_B$$

$$\text{b) } \dot{\beta} = u_B \sin \psi_B - u_A \sin \psi_A + v_B \cos \psi_B - v_A \cos \psi_A \quad (2-10)$$

Since ψ_A and ψ_B are small, $\cos \psi_A \approx 1.0$ and $\cos \psi_B \approx 1.0$.

And since $v_A \ll u_A$ and $v_B \ll u_B$, and $\sin \psi_A$ and $\sin \psi_B$ are small, the products $v_A \sin \psi_A$ and $v_B \sin \psi_B$ are second order. Thus

(2-10), a), becomes:

$$\dot{\alpha} \approx u_B - u_A = ((u_B)_0 + \Delta u_B) - ((u_A)_0 + \Delta u_A)$$

where $(u_B)_0 = u_0 = (u_A)_0$, and:

$$\dot{\alpha} \approx \Delta u_B - \Delta u_A$$

Also, since ψ_A and ψ_B are small, $\sin\psi_A \approx \psi_A$ and $\sin\psi_B \approx \psi_B$. Thus, (2-10), b), becomes:

$$\begin{aligned}\dot{\beta} &\approx u_B \psi_B - u_A \psi_A + \Delta v_B - \Delta v_A \\ &\approx \psi_B (u_B)_0 - \psi_A (u_A)_0 + \underbrace{\psi_B \Delta u_B - \psi_A \Delta u_A}_{\text{2nd order}} + \Delta v_B - \Delta v_A\end{aligned}$$

Therefore, the equations, (2-10), become:

$$a) \dot{\alpha} \approx \Delta u_B - \Delta u_A$$

$$b) \dot{\beta} \approx u_{0B} \Delta \psi_B - u_{0A} \Delta \psi_A + \Delta v_B - \Delta v_A \quad (2-11)$$

≈ 0

IX. The Complete Set of Linear Equations of Motion for the Two Ship System

Incorporating all the foregoing discussion, the linear equations of motion for the two-ship system emerge:

Ship A: (2-12)

$$\begin{aligned}a) & X_{\alpha_A} \Delta \alpha + X_{\beta_A} \Delta \beta + X_{u_A} \Delta u_A + X_{\dot{u}_A} \dot{u}_A + X_{v_A} \Delta v_A + X_{\dot{v}_A} \dot{v}_A + X_{\psi_A} \dot{\psi}_A \\ & + X_{\dot{\psi}_A} \dot{\psi}_A + X_{\delta_A} [k_{1A} + (k_{2A} - k_{1A} \Delta t_{1A}) \mathcal{D} - k_{2A} \Delta t_{1A} \mathcal{D}^2] \Delta \psi_A \\ & + X_{\delta_A} [k_{3A} + (k_{4A} - k_{3A} \Delta t_{1A}) \mathcal{D} - k_{4A} \Delta t_{1A} \mathcal{D}^2] \Delta \beta \\ & + X_{n_A} [k_{5A} + (k_{6A} - k_{5A} \Delta t_{2A}) \mathcal{D} - k_{6A} \Delta t_{2A} \mathcal{D}^2] \Delta \alpha - m_A \dot{u}_A = 0\end{aligned}$$

$$b) \quad Y_{\alpha A} \Delta \alpha + Y_{\beta A} \Delta \beta + Y_{u A} \Delta u_A + Y_{\dot{u}_A} \dot{u}_A + Y_{v A} \Delta v_A + Y_{\dot{v}_A} \dot{v}_A + Y_{\dot{\psi}_A} \dot{\psi}_A$$

$$+ Y_{\ddot{\psi}_A} \ddot{\psi}_A + Y_{\delta A} [k_{1A} + (k_{2A} - k_{1A} \Delta t_{1A}) \mathcal{B} - k_{2A} \Delta t_{1A} \mathcal{B}^2] \Delta \psi_A$$

$$+ Y_{\delta A} [k_{3A} + (k_{4A} - k_{3A} \Delta t_{1A}) \mathcal{B} - k_{4A} \Delta t_{1A} \mathcal{B}^2] \Delta \beta$$

$$+ Y_{n_A} [k_{5A} + (k_{6A} - k_{5A} \Delta t_{2A}) \mathcal{B} - k_{6A} \Delta t_{2A} \mathcal{B}^2] \Delta \alpha$$

$$- m_A [\dot{v}_A + u_{0A} \dot{\psi}_A + x_{g_A} \ddot{\psi}_A] = 0$$

$$c) \quad N_{\alpha A} \Delta \alpha + N_{\beta A} \Delta \beta + N_{u A} \Delta u_A + N_{\dot{u}_A} \dot{u}_A + N_{v A} \Delta v_A + N_{\dot{v}_A} \dot{v}_A + N_{\dot{\psi}_A} \dot{\psi}_A$$

$$+ N_{\ddot{\psi}_A} \ddot{\psi}_A + N_{\delta A} [k_{1A} + (k_{2A} - k_{1A} \Delta t_{1A}) \mathcal{B} - k_{2A} \Delta t_{1A} \mathcal{B}^2] \Delta \psi_A$$

$$+ N_{\delta A} [k_{3A} + (k_{4A} - k_{3A} \Delta t_{1A}) \mathcal{B} - k_{4A} \Delta t_{1A} \mathcal{B}^2] \Delta \beta$$

$$+ N_{n_A} [k_{5A} + (k_{6A} - k_{5A} \Delta t_{2A}) \mathcal{B} - k_{6A} \Delta t_{2A} \mathcal{B}^2] \Delta \alpha$$

$$- I_{z_A} \ddot{\psi}_A - m_A x_{g_A} (\dot{v}_A + u_{0A} \dot{\psi}_A) = 0$$

$$\left. \begin{array}{l} d) \\ e) \\ f) \end{array} \right\} \quad \text{Precisely Similar Equations for Ship B.}$$

$$g) \quad \dot{\alpha} = \Delta u_B - \Delta u_A = \mathcal{B} \alpha$$

$$h) \quad \dot{\beta} = u_{0A} \Delta \psi_B - u_{0A} \Delta \psi_A + \Delta v_B - \Delta v_A = \mathcal{B} \beta$$

X. Identifying Zero-Value Derivatives

Arguing from symmetry of the ship about its centerline plane, Abkowitz demonstrates that X_v , the change in x-direction forces of the ship caused by a change in v , with all other variables fixed at their equilibrium values, must have a value of zero. Similarly, it is demonstrated that $X_{\dot{v}} = X_{\dot{\psi}} = X_{\ddot{\psi}} = 0$. ([A-3], pp. I-28 and I-30). Due to the port and starboard symmetry of the rudder, X_{δ} is also zero. ([A-3], pg. I-67).

Further arguments from symmetry demonstrate that in the open water case, Y_u , $Y_{\dot{u}}$, N_u and $N_{\dot{u}}$ are all zero. But for the two-ship system of interest, the interaction side force and yaw moment are affected by changes in speed. Thus Y_u and N_u cannot be assumed to be zero. The effect on this force and moment, of a small acceleration, is, however, still assumed negligible, and $Y_{\ddot{u}}$ and $N_{\ddot{u}}$ are assumed to be zero.

On examining the hydrodynamic derivatives due to the orientation of the ships with respect to each other, further identification of negligible derivatives can be made. The derivatives X_{α} and X_{β} , representing, respectively, the change in x-direction force caused by small changes in longitudinal and lateral separations of the ships, will, at most, be small. Experiments have shown that interaction phenomena, while affecting Y forces and N moments, have no measurable effect on the forward motion of the ships, or on their resistance forces. ([A-1], [A-2]) Thus, X_{α} and X_{β} are set equal to zero.

Summarizing:

$$X_v = X_{\dot{v}} = X_{\dot{\psi}} = X_{\dot{\psi}} = X_{\delta} = X_{\alpha} = X_{\beta} = Y_{\dot{u}} = N_{\dot{u}} = 0 \quad (2-13)$$

With this information, the equations of motion are simplified to:

Ship A: (2-14)

$$a) X_{u_A} \Delta u_A + X_{\dot{u}_A} \dot{u}_A + X_{n_A} [k_{5_A} + (k_{6_A} - k_{5_A} \Delta t_{2_A}) \mathcal{J}]$$

$$-k_{6_A} \Delta t_{2_A} \mathcal{J}^2] \Delta \alpha - m_A \dot{u}_A = 0$$

$$b) (Y_{\alpha_A} \Delta \alpha + Y_{\beta_A} \Delta \beta + Y_{u_A} \Delta u_A + Y_{v_A} \Delta v_A + Y_{\dot{v}_A} \dot{v}_A + Y_{\dot{\psi}_A} \dot{\psi}_A$$

$$+ Y_{\ddot{\psi}_A} \ddot{\psi}_A + Y_{\delta_A} [k_{1_A} + (k_{2_A} - k_{1_A} \Delta t_{1_A}) \mathcal{J} - k_{2_A} \Delta t_{1_A} \mathcal{J}^2] \Delta \psi_A$$

$$+ Y_{\delta_A} [k_{3_A} + (k_{4_A} - k_{3_A} \Delta t_{1_A}) \mathcal{J} - k_{4_A} \Delta t_{1_A} \mathcal{J}^2] \Delta \beta$$

$$+ Y_{n_A} [k_{5_A} + (k_{6_A} - k_{5_A} \Delta t_{2_A}) \mathcal{J} - k_{6_A} \Delta t_{2_A} \mathcal{J}^2] \Delta \alpha$$

$$- m_A (\dot{v}_A + u_A \dot{\psi}_A + x_{g_A} \ddot{\psi}_A) = 0$$

$$c) \quad N_{\alpha_A} \Delta \alpha + N_{\beta_A} \Delta \beta + N_{u_A} \Delta u + N_{v_A} \Delta v + N_{\dot{v}_A} \dot{v}_A + N_{\dot{\psi}_A} \dot{\psi}_A$$

$$+ N_{\ddot{\psi}_A} \ddot{\psi}_A + N_{\delta_A} [k_{1A} + (k_{2A} - k_{1A} \Delta t_{1A}) \mathcal{D} - k_{2A} \Delta t_{1A} \mathcal{D}^2] \Delta \psi_A$$

$$+ N_{\delta_A} [k_{3A} + (k_{4A} - k_{3A} \Delta t_{1A}) \mathcal{D} - k_{4A} \Delta t_{1A} \mathcal{D}^2] \Delta \beta$$

$$+ N_{n_A} [k_{5A} + (k_{6A} - k_{5A} \Delta t_{2A}) \mathcal{D} - k_{6A} \Delta t_{2A} \mathcal{D}^2] \Delta \alpha$$

$$- I_{z_A} \ddot{\psi}_A - m_A \times g_A (\dot{v}_A + u_O \dot{\psi}_A) = 0$$

$$\left. \begin{array}{l} d) \\ e) \\ f) \end{array} \right\} \quad \text{Precisely Similar Equations for Ship B.}$$

$$g) \quad \mathcal{D} \alpha = \Delta u_B - \Delta u_A$$

$$h) \quad \mathcal{D} \beta = u_O \Delta \psi_B - u_O \Delta \psi_A + \Delta v_B - \Delta v_A$$

XI. Rearranging Equations of Motion

At this point, terms are gathered so as to have each variable multiplied by the sum of all its coefficients in each equation.

Remembering that $\mathcal{D}(\text{variable}) = \partial(\text{variable})/\partial t$, then:

$$\mathcal{D}(\Delta u) = \frac{\partial(\Delta u)}{\partial t} = \frac{\partial(u - u_O)}{\partial t} = \frac{\partial u}{\partial t} - \frac{\partial u_O}{\partial t} \quad (2-15)$$

But u_O is constant, so $\mathcal{D}(\Delta u) = \mathcal{D}u$.

Thus:

Ship A:

(2-16)

$$a) [X_{n_A} k_{5_A} + X_{n_A} (k_{6_A} - k_{5_A} \Delta t_{2_A}) \mathcal{D} - X_{n_A} k_{6_A} \Delta t_{2_A} \mathcal{D}^2] \Delta \alpha$$

$$+ [X_{u_A} + (X_{\dot{u}_A} - m_A) \mathcal{D}] \Delta u_A = 0$$

$$b) [(Y_{\alpha_A} + Y_{n_A} k_{5_A}) + Y_{n_A} (k_{6_A} - k_{5_A} \Delta t_{2_A}) \mathcal{D} - Y_{n_A} k_{6_A} \Delta t_{2_A} \mathcal{D}^2] \Delta \alpha$$

$$+ [(Y_{\beta_A} + Y_{\delta_A} k_{3_A}) + Y_{\delta_A} (k_{4_A} - k_{3_A} \Delta t_{1_A}) \mathcal{D} - Y_{n_A} k_{4_A} \Delta t_{1_A} \mathcal{D}^2] \Delta \beta$$

$$+ [Y_{u_A}] \Delta u_A + [Y_{v_A} + (Y_{\dot{v}_A} - m_A) \mathcal{D}] \Delta v_A$$

$$+ [Y_{\delta_A} k_{1_A} + ((Y_{\dot{\psi}_A} - m_A u_O) + Y_{\delta_A} (k_{2_A} - k_{1_A} \Delta t_{1_A})) \mathcal{D}$$

$$+ ((Y_{\dot{\psi}_A} - m_A x_{g_A}) - Y_{\delta_A} k_{2_A} \Delta t_{1_A}) \mathcal{D}^2] \Delta \psi_A = 0$$

$$c) [(N_{\alpha_A} + N_{n_A} k_{5_A}) + N_{n_A} (k_{6_A} - k_{5_A} \Delta t_{2_A}) \mathcal{D} - N_{n_A} k_{6_A} \Delta t_{2_A} \mathcal{D}^2] \Delta \alpha$$

$$+ [(N_{\beta_A} + N_{\delta_A} k_{3_A}) + N_{\delta_A} (k_{4_A} - k_{3_A} \Delta t_{1_A}) \mathcal{D} - N_{\delta_A} k_{4_A} \Delta t_{1_A} \mathcal{D}^2] \Delta \beta$$

$$+ N_{u_A} \Delta u_A + [N_{v_A} + (N_{\dot{v}_A} - m_A x_{g_A}) \mathcal{D}] \Delta v_A + [N_{\delta_A} k_{1_A} + ((N_{\dot{\psi}_A} - m_A x_{g_A} u_O)$$

$$+ N_{\delta_A} (k_{2_A} - k_{1_A} \Delta t_{1_A})) \mathcal{D} + ((N_{\dot{\psi}_A} - I_{z_A}) - N_{\delta_A} k_{2_A} \Delta t_{1_A}) \mathcal{D}^2] \Delta \psi_A = 0$$

In writing the equations for Ship B use is made of the relationships provided by equation g and h of (2-14), to substitute for Δu_B and Δv_B in the Ship B equations. From these equations, $\Delta u_B = \Delta u_A + \mathcal{D}\alpha$ and

$$\Delta v_B = \mathcal{D}\beta + \Delta v_A + u_O \Delta \psi_A - u_O \Delta \psi_B. \quad \text{Thus:}$$

Ship B:

(2-16)

$$d) [X_{n_B} k_{5_B} + (X_{n_B} (k_{6_B} - k_{5_B} \Delta t_{2_B}) + X_{u_B}) \mathcal{D}$$

$$+ ((X_{\dot{u}_B} - m_B) - X_{n_B} k_{6_B} \Delta t_{2_B}) \mathcal{D}^2] \Delta \alpha$$

$$+ [X_{u_B} + (X_{\dot{u}_B} - m_B) \mathcal{D}] \Delta u_A = 0$$

$$e) [(Y_{\alpha_B} + Y_{n_B} k_{5_B}) + (Y_{n_B} (k_{6_B} - k_{5_B} \Delta t_{2_B}) + Y_{u_B}) \mathcal{D} - Y_{n_B} k_{6_B} \Delta t_{2_B} \mathcal{D}^2] \Delta \alpha$$

$$+ [(Y_{\beta_B} + Y_{\delta_B} k_{3_B}) + (Y_{\delta_B} (k_{4_B} - k_{3_B} \Delta t_{1_B}) + Y_{v_B}) \mathcal{D}$$

$$+ ((Y_{\dot{v}_B} - m_B) - Y_{\delta_B} k_{4_B} \Delta t_{1_B}) \mathcal{D}^2] \Delta \beta + [Y_{u_B}] \Delta u_A + [Y_{v_B} + (Y_{\dot{v}_B} - m_B) \mathcal{D}] \Delta v_A$$

$$+ [u_O Y_{v_B} + u_O (Y_{\dot{v}_B} - m_B) \mathcal{D}] \Delta \psi_A$$

$$+ [(Y_{\delta_B} k_{1_B} - u_O Y_{v_B}) + ((Y_{\dot{\psi}_B} - m_B u_O) + Y_{\delta_B} (k_{2_B} - k_{1_B} \Delta t_{1_B})$$

$$- u_O (Y_{\dot{v}_B} - m_B)) \mathcal{D} + ((Y_{\dot{\psi}_B} - m_B x_{g_B}) - Y_{\delta_B} k_{2_B} \Delta t_{1_B}) \mathcal{D}^2] \Delta \psi_B = 0$$

$$\begin{aligned}
& f) [(N_{\alpha_B} + N_{n_B} k_{5_B}) + (N_{n_B} (k_{6_B} - k_{5_B} \Delta t_{2_B}) + N_{u_B}) \mathcal{D} - N_{n_B} k_{6_B} \Delta t_{2_B} \mathcal{D}^2] \Delta \alpha \\
& + [(N_{\beta_B} + N_{\delta_B} k_{3_B}) + (N_{\delta_B} (k_{4_B} - k_{3_B} \Delta t_{1_B}) + N_{v_B}) \mathcal{D} + (N_{\dot{v}_B} - m_B x_{g_B} \\
& - N_{\delta_B} k_{4_B} \Delta t_{1_B}) \mathcal{D}^2] \Delta \beta + [N_{u_B}] \Delta u_A + [N_{v_B} + (N_{\dot{v}_B} - m_B x_{g_B}) \mathcal{D}] \Delta v_A \\
& + [u_{o_{v_B}} + u_{o_{\dot{v}_B}} (N_{\dot{v}_B} - m_B x_{g_B}) \mathcal{D}] \Delta \psi_A + [(N_{\delta_B} k_{1_B} - u_{o_{v_B}}) \\
& + (N_{\dot{\psi}_B} - m_B x_{g_B} u_{o_{\dot{\psi}_B}}) + N_{\delta_B} (k_{2_B} - k_{1_B} \Delta t_{1_B}) - u_{o_{\dot{v}_B}} (N_{\dot{v}_B} - m_B x_{g_B})] \mathcal{D} \\
& + ((N_{\dot{\psi}_B} - I_{z_B}) - N_{\delta_B} k_{2_B} \Delta t_{1_B}) \mathcal{D}^2] \Delta \psi_B = 0
\end{aligned}$$

These, then, are the six equations of motion in six variables, in their final form for solution. The equations are of the form: (2-17)

$$\begin{aligned}
(AX1) \Delta \alpha + 0 + (AX3) \Delta u_A + 0 + 0 + 0 &= 0 \\
(AY1) \Delta \alpha + (AY2) \Delta \beta + (AY3) \Delta u_A + (AY4) \Delta v_A + (AY5) \Delta \psi_A + 0 &= 0 \\
(AN1) \Delta \alpha + (AN2) \Delta \beta + (AN3) \Delta u_A + (AN4) \Delta v_A + (AN5) \Delta \psi_A + 0 &= 0 \\
(BX1) \Delta \alpha + 0 + (BX3) \Delta u_A + 0 + 0 + 0 &= 0 \\
(BY1) \Delta \alpha + (BY2) \Delta \beta + (BY3) \Delta u_A + (BY4) \Delta v_A + (BY5) \Delta \psi_A + (BY6) \Delta \psi_B &= 0 \\
(BN1) \Delta \alpha + (BN2) \Delta \beta + (BN3) \Delta u_A + (BN4) \Delta v_A + (BN5) \Delta \psi_A + (BN6) \Delta \psi_B &= 0
\end{aligned}$$

This is a set of six simultaneous, linear differential equations in six unknowns. The term BX3, for instance, is used to denote: Ship B, X-equation, 3rd term, (or Δu_A term). Each of the coefficients is, itself, a polynomial in \mathcal{D} , as

can be seen from equations (2-16), for which (2-17) is a shorthand notation.

XII. Solving the Equations of Motion

If the terms AX_1 , BX_1 , etc., were just numerical coefficients, straight-forward algebraic techniques could be used to solve this system of equations. Yet, while these coefficients are, themselves, polynomials in the operator \mathcal{D} , Abkowitz shows that \mathcal{D} can be treated as an algebraic quantity. ([A-3], pp. I-26, 27). This is possible because the hydrodynamic derivatives are defined as the slope of a force or moment versus a dynamic variable taken at the equilibrium condition. Thus the terms in the coefficients AX_1 , etc., other than \mathcal{D} , are constants in time.

The solution for any of the variables is, then, the determinant of the coefficients, with the column corresponding to the variable being solved for replaced by the right-hand side column vector, divided by the determinant of all the coefficients.

For example:

(see next page)

$$\Delta\alpha = \frac{\begin{vmatrix} 0 & 0 & AX3 & 0 & 0 & 0 \\ 0 & AY2 & AY3 & AY4 & AY5 & 0 \\ 0 & AN2 & AN3 & AN4 & AN5 & 0 \\ 0 & 0 & BX3 & 0 & 0 & 0 \\ 0 & BY2 & BY3 & BY4 & BY5 & BY6 \\ 0 & BN2 & BN3 & BN4 & BN5 & BN6 \end{vmatrix}}{\begin{vmatrix} AX1 & 0 & AX3 & 0 & 0 & 0 \\ AY1 & AY2 & AY3 & AY4 & AY5 & 0 \\ AN1 & AN2 & AN3 & AN4 & AN5 & 0 \\ BX1 & 0 & BX3 & 0 & 0 & 0 \\ BY1 & BY2 & BY3 & BY4 & BY5 & BY6 \\ BN1 & BN2 & BN3 & BN4 & BN5 & BN6 \end{vmatrix}} = \frac{0}{|DET|} \quad (2-18)$$

Evaluation of DET leads to a polynomial in \mathcal{D} , (made up of sums of products of polynomials in \mathcal{D} , such as AX1, etc.), whose coefficients are various combination of the hydrodynamic derivatives and control system constants. Thus the solution for each variable would be:

$$\text{variable} = \frac{0}{a_1 \mathcal{D}^n + a_2 \mathcal{D}^{n-1} + \dots + a_{n+1}} \quad (2-19)$$

If DET is other than zero, the solution for all variables would be identically zero for all time after a small disturbance from equilibrium---a physical impossibility. Thus, the denominator, the determinant DET, must be set equal to zero. Setting this polynomial in \mathcal{D} , resulting from the evaluation

of DET, equal to zero, the roots of the polynomial can be obtained, where $\sigma_1, \sigma_2, \dots, \sigma_n$ are these roots. Then:

$$\text{variable} = \frac{0}{(\mathcal{D} - \sigma_n)(\mathcal{D} - \sigma_{n-1}) \dots (\mathcal{D} - \sigma_1)} \quad (2-20)$$

Abkowitz demonstrates how the operation $\frac{1}{(\mathcal{D} - \sigma)} [z]$ is equivalent to:

$$\frac{1}{(\mathcal{D} - \sigma)} [z] = e^{\sigma t} \int e^{-\sigma t} [z] dt \quad (2-21)$$

$$\text{Thus, } \frac{0}{(\mathcal{D} - \sigma)} = e^{\sigma t} \int e^{-\sigma t} (0) dt = c_1 e^{\sigma t}$$

Carrying out this operation successively to evaluate equation (2-20) yields:

$$\text{variable} = c_1 e^{\sigma_1 t} + c_2 e^{\sigma_2 t} \dots c_n e^{\sigma_n t}$$

where the c_i are arbitrary constants of integration.

XIII. Establishing the Stability Criterion for the System

Since each of the variables is a sum of exponential terms in time, it is clear that the only way the variables can all go to zero as time increases, is for all the σ_i (or all real parts where the σ_i are complex) to be negative. The criterion of stability of the two ship system is for all the variables, representing, as they do, deviations from the equilibrium condition, to go to zero as time increases. For when all the variables have gone to zero, the system has regained the equilibrium condition, meeting the definition of a stable system.

Thus, the stability of the system requires that the real parts of all σ_i be negative. As the σ_i are the roots of the characteristic equation of stability resulting from setting the polynomial in \mathcal{D} , (which in turn results from evaluating DET), equal to zero, the stability requirement for the two-ship system is that the real parts of all the roots of DET be less than zero.

Chapter III

Procedure

I. Background

As stated at the conclusion of Chapter I, a modified OLNA/MARINER class merchant hull is the ship used in the mathematical model. As indicated there, this was done because open water hydrodynamic derivatives, in non-dimensional form, were available for a MARINER, ([A-4], [A-5]), and because the only good, quantitative interaction phenomena data available were for a ship quite similar to a MARINER---the R.F.A. OLNA.

II. Open Water Hydrodynamic Derivatives

Reference A-5 presents experimentally determined values for most open water derivatives, measured with a planar motion mechanism, for a MARINER model. Reference A-4 utilized these values, with the exception of X_u , for which a different value was determined. The accuracy of the values used in [A-4] was borne out with full scale sea trials and these values are used in this thesis for the open water derivatives, for ship A, and are listed in Table III-1.

III. Obtaining the Interaction Hydrodynamic Derivatives

A. Y_α , Y_β , N_α , N_β .

Values for hydrodynamic derivatives, as such, arising from interaction effects, could not be obtained. The interaction data for OLNA consisted of dimensionless forces and moments at side-to-side ship separations of 50 and 100 feet,

for all longitudinal positions of the midships sections of the ships between -600 and +600 feet. Also available were force and moment data for OLNA at side-to-side ship separations of from 30 to 150 feet, with the ships abreast. [A-1] (See Figures I-3, I-4 and I-5)

Using the data illustrated by Figures I-3, 4 and 5, the only positions at which Y_α , Y_β , N_α and N_β could all be determined were at side-to-side distances of 50 and 100 feet, with the ships abreast. (Points 1 and 6, Figure III-1). This limitation of the examination of system stability to only two ship positions was felt to be too restrictive. As the only means available for obtaining the desired derivatives at other positions of interest was extrapolation and interpolation of the OLNA data, this course was followed for both Y force and N moment. As data was available at side-to-side distances (STSD, for short), of 50 and 100 feet, these values were chosen as bounds to the range of interest, just as values of -600 and +600 feet were chosen as bounds in the longitudinal direction. The way in which Y force was interpolated in the range of interest will be illustrated.

The Y force was known along three lines of position, as shown in Figures I-3, and I-5, and as in Figure III-1. To determine Y at positions other than those known in Figure III-1, it was assumed that the ratio of change in Y between 1' and 2' to the change between 1' and 6' was the same as the ratio of the change in Y between 1 and 2 to the change between 1 and 6.

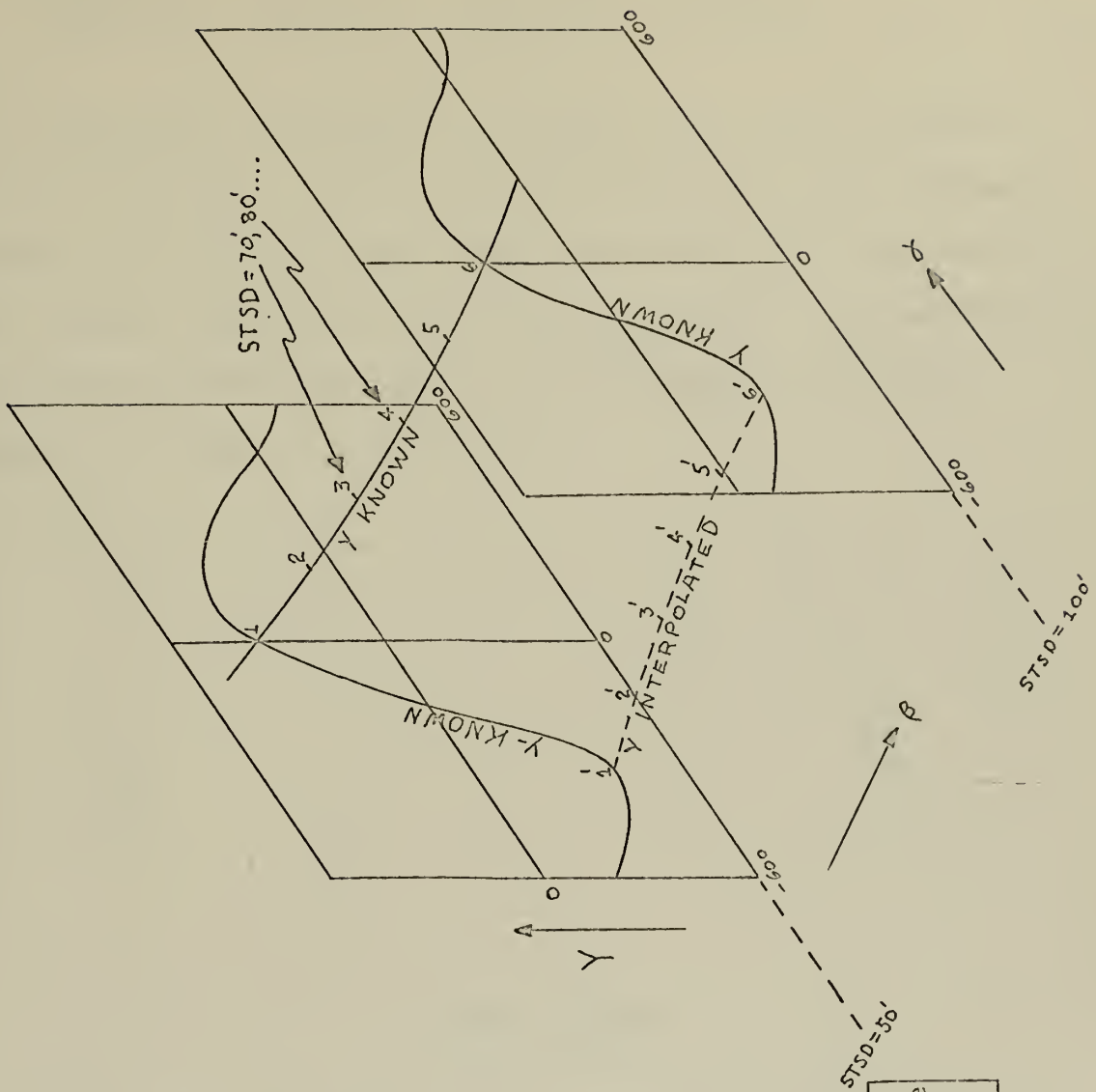


Illustration of
Interpolation Technique
Used for Y Force and
N Moment

Figure III-1

Or:

$$\frac{(Y_2 - Y_1)}{(Y_6 - Y_1)} = \frac{(Y_2 - Y_1)}{(Y_6 - Y_1)} ; \frac{(Y_3 - Y_1)}{(Y_6 - Y_1)} = \frac{(Y_3 - Y_1)}{(Y_6 - Y_1)} , \text{ etc.} \quad (3-1)$$

The points 1 through 6 in Figure III-1 were chosen at 10 foot intervals, and the interpolation for these values of STSD=60, 70, 80 and 90 feet was performed at 50 foot intervals in α . Thus, values of Y were obtained at all intersections of a grid with lateral spacing of 10 feet and longitudinal spacing of 50 feet, as shown below:

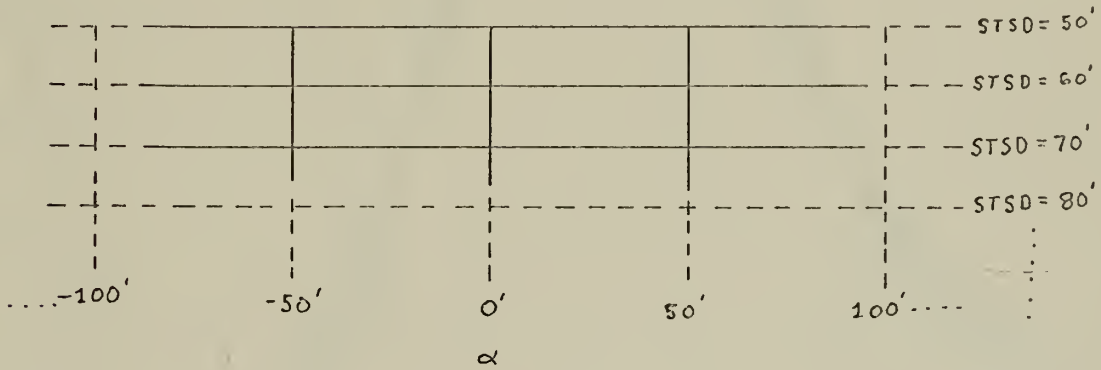
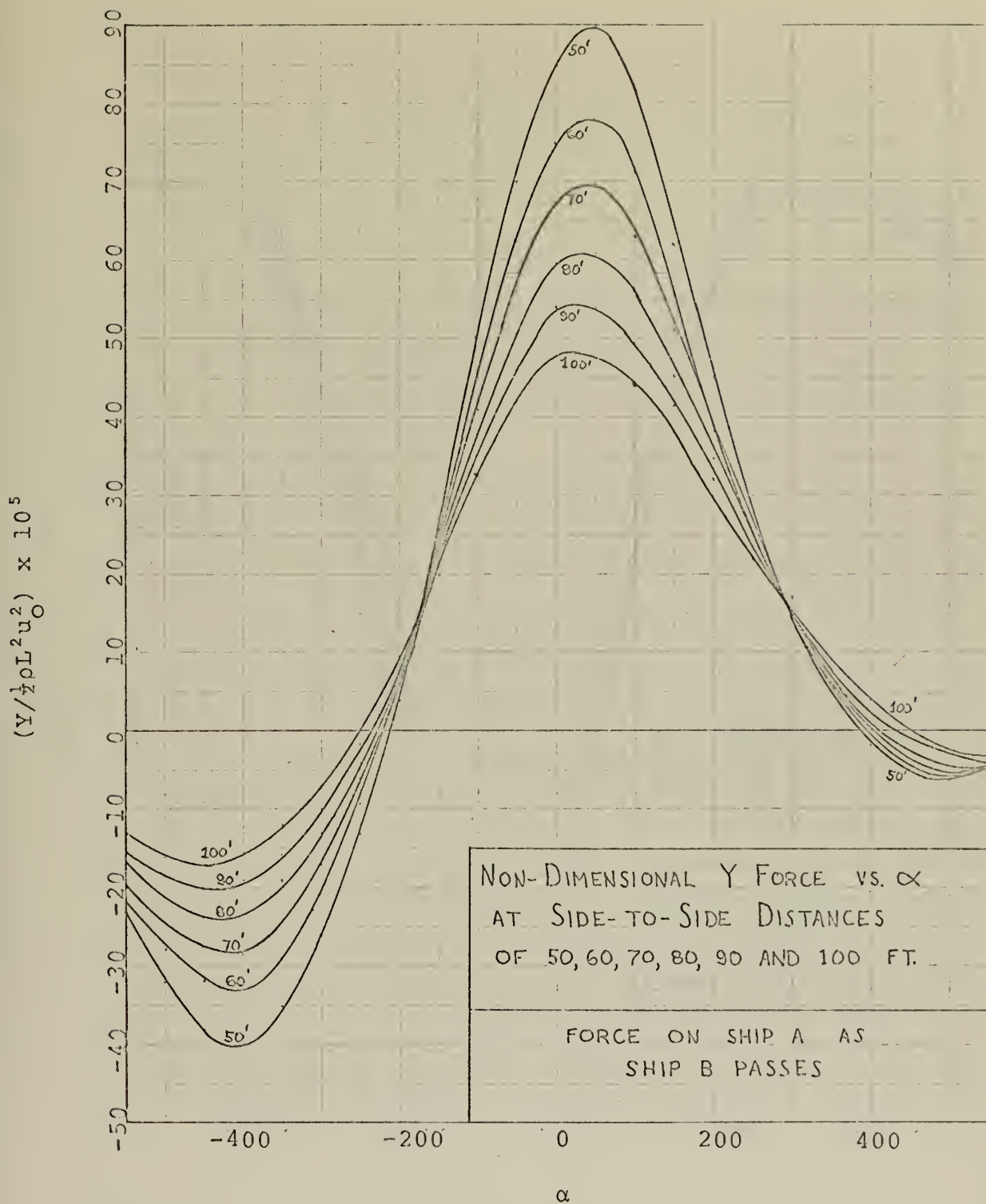


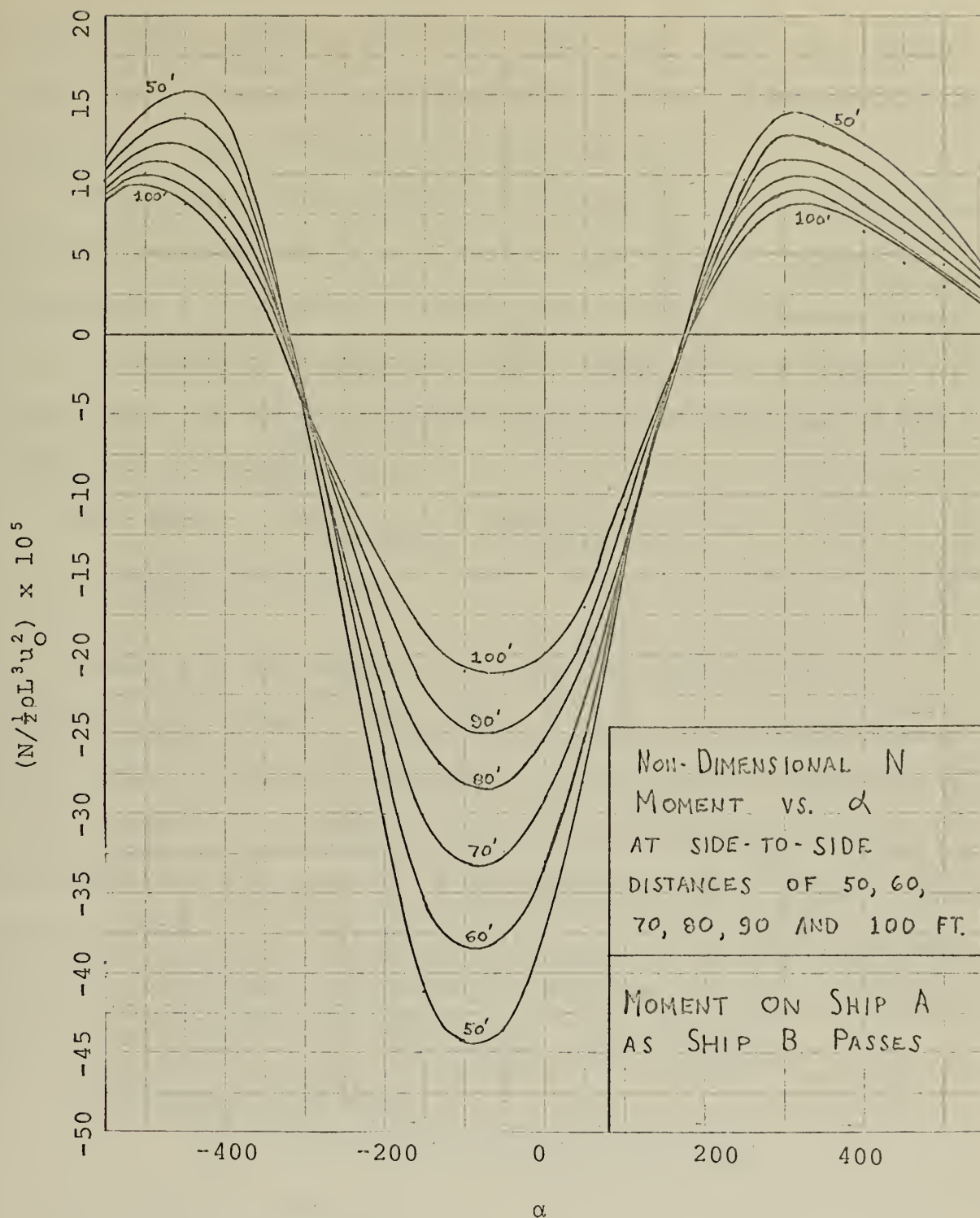
Figure III-2

The results of such interpolations for both Y and N are shown as figures III-3 and III-4, where the curves for STSD = 50 and 100 feet are from [A-1] and the others are interpolated. The interpolated curves are felt to give a reasonable picture of the way the interaction force and moment would be expected to vary as STSD is changed. And, of course, the points on the line $\alpha=0$ are the same points as those plotted in Figure I-5.



Dimensionless Y Force vs. Longitudinal Separation

FIGURE III-3



Dimensionless N Moment vs. Longitudinal Separation

FIGURE III-4

Obtaining the interaction forces and moments over this range of $50' \leq \text{STSD} \leq 100'$ and $-600' \leq \alpha \leq +600'$ is, though, just a step along the way to obtaining the derivatives Y_α , Y_β , N_α and N_β , which are what is really desired.

The longitudinal separation parameter, α , is represented directly on the curves of Y and N. The lateral separation parameter, β , is related to the side-to-side distances shown on the figures by a constant, for a given pair of ships. Since β is the lateral separation of the centerlines of the ships, it is apparent that:

$$\begin{aligned}\beta &= \text{STSD} + (\text{BEAM}_{\text{Ship A}} + \text{BEAM}_{\text{Ship B}})/2 \\ &= \text{STSD} + \text{Constant}\end{aligned}\tag{3-2}$$

Thus, for two ships of 50' beam, an STSD of 50' is equivalent to $\beta=100'$. Therefore, Figure III-3, for example, represents a surface in space, (as sketched in Figure III-1), whose horizontal coordinates are α and β and whose vertical coordinate is the value of Y everywhere on the surface. Then, to get $\partial Y/\partial \alpha$, or Y_α , at any point, the slope of the Y surface in the α direction, at the point, must be obtained. Similarly, Y_β is the slope in the β direction. For example:

$$\begin{aligned}\text{a) } Y(\alpha, \beta) &= Y(100, 60) \\ &= ((Y(150, 60) - Y(50, 60)) / (150 - 50)) \\ \text{b) } Y(\alpha, \beta) &= Y(100, 60) \\ &= ((Y(100, 70) - Y(100, 50)) / (70 - 50))\end{aligned}\tag{3-3}$$

In this way, Y_α everywhere in the range $(50' \leq \text{STSD} \leq 100')$;

$-550' \leq \alpha \leq 550'$), was obtained, as was Y_β everywhere in the range ($60' \leq \text{STSD} \leq 90'$; $-600' \leq \alpha \leq 600'$). (The values of Y_α and Y_β at the extremities of the range of known Y values, in the direction of differentiation, could not, of course, be obtained by this simple procedure).

The values of N_α and N_β over the same ranges were obtained in the same way.

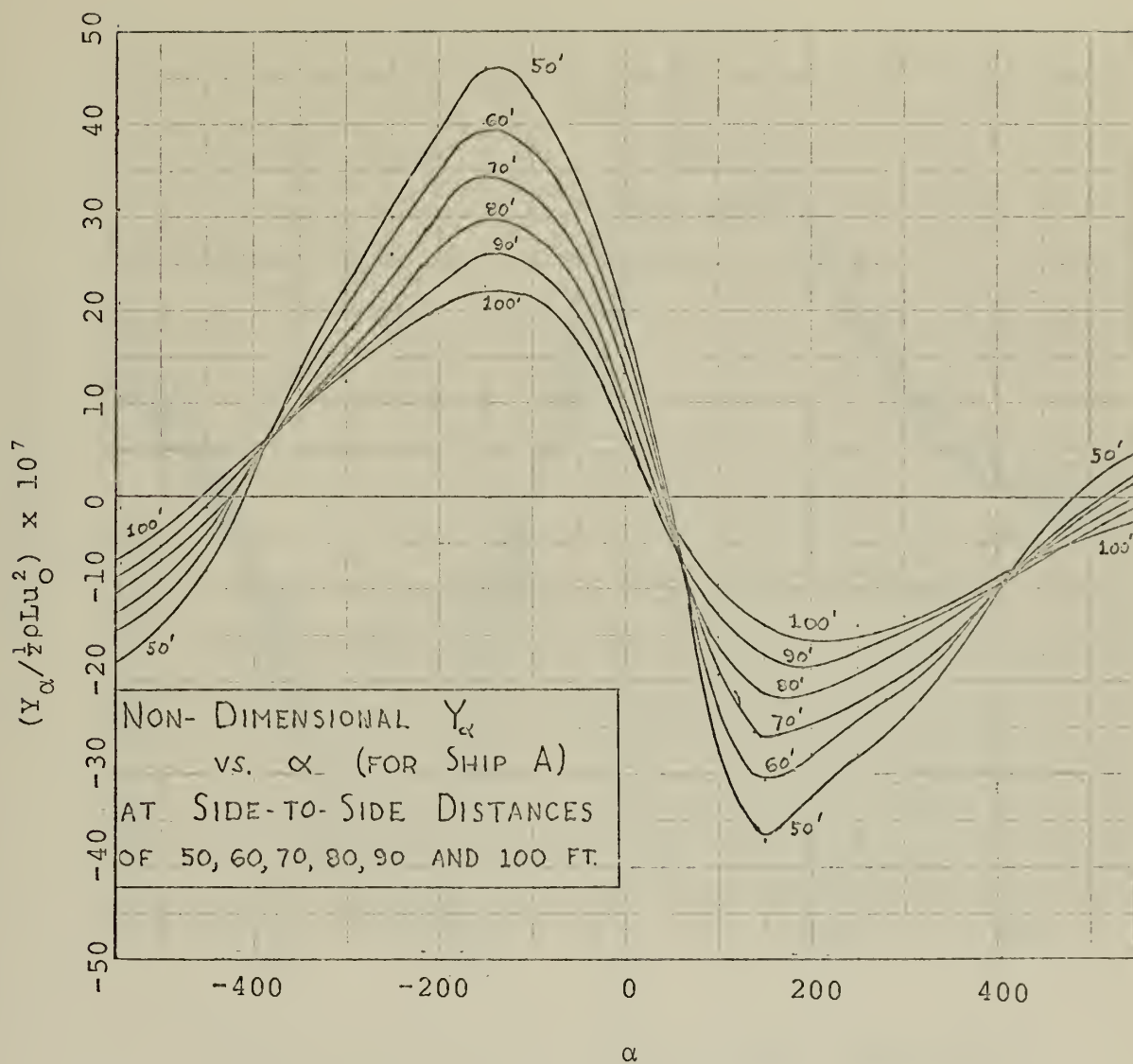
A computer program, program INTCOF, was written to take the values of Y and N at the locations known from Figures I-3, 4 and 5, to interpolate to get Y and N throughout the range, and then determine Y_α , Y_β , N_α and N_β in this range. The curves in Figures III-3 and III-4 for side-to-side distances of 60, 70, 80 and 90 feet are plots of the Y and N outputs of the INTCOF interpolations.

Similarly, the curves in Figures III-5 and III-6 for STSD 60, 70, 80 and 90 feet are INTCOF outputs. And in Figures III-7 and III-8, the curves are all INTCOF outputs.

Program INTCOF is described in detail in Appendix II, and the results of the program for the modified OLNA/MARINER ship are included therein.

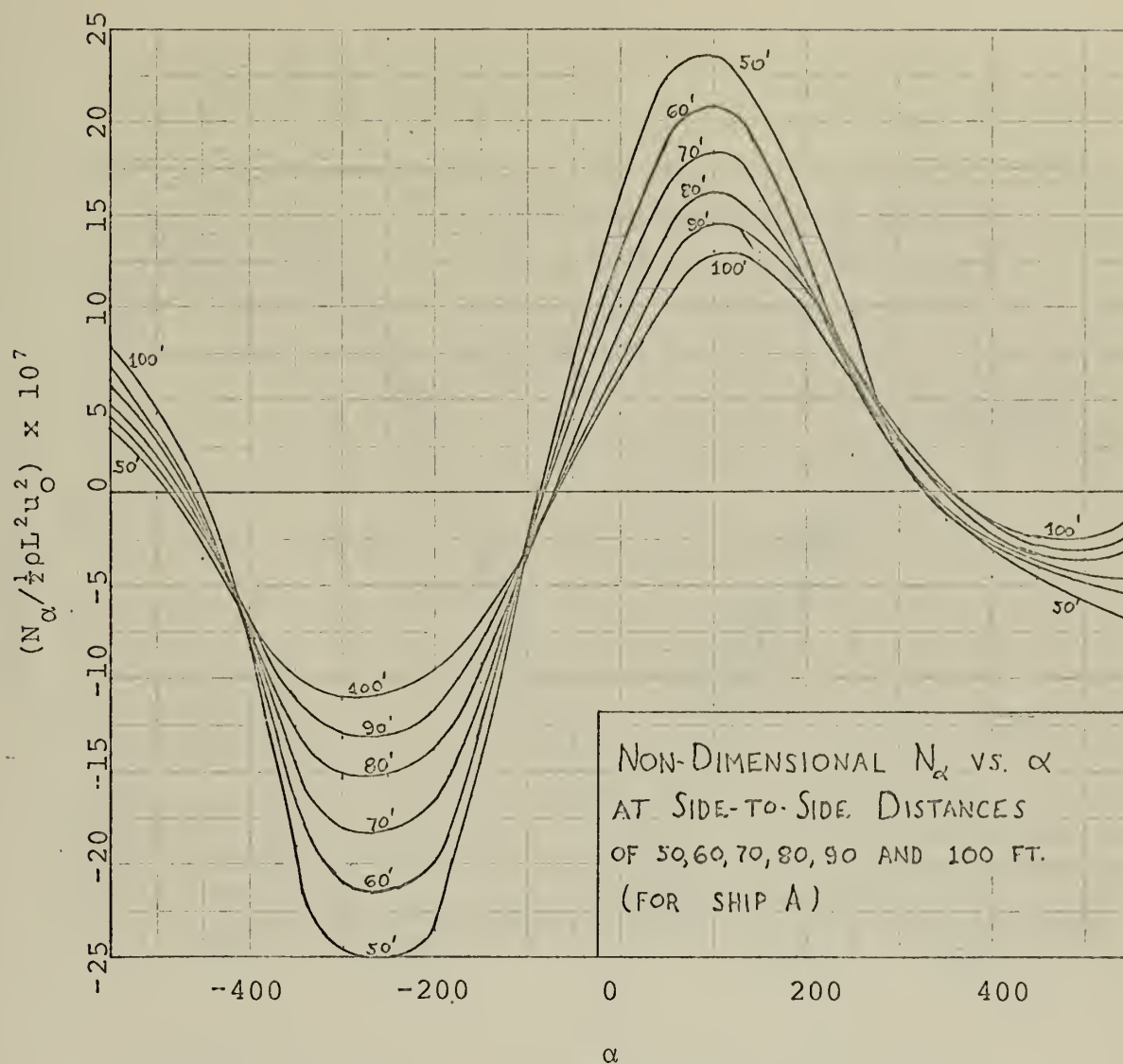
B. Y_u , N_u .

As noted in Chapter I, experimental investigators in the field of ship interactions have noted that interaction phenomena vary with speed in the same way as the resistance of the ship---or that interaction effects vary as the square of the speed. The OLNA data used for interaction effects was



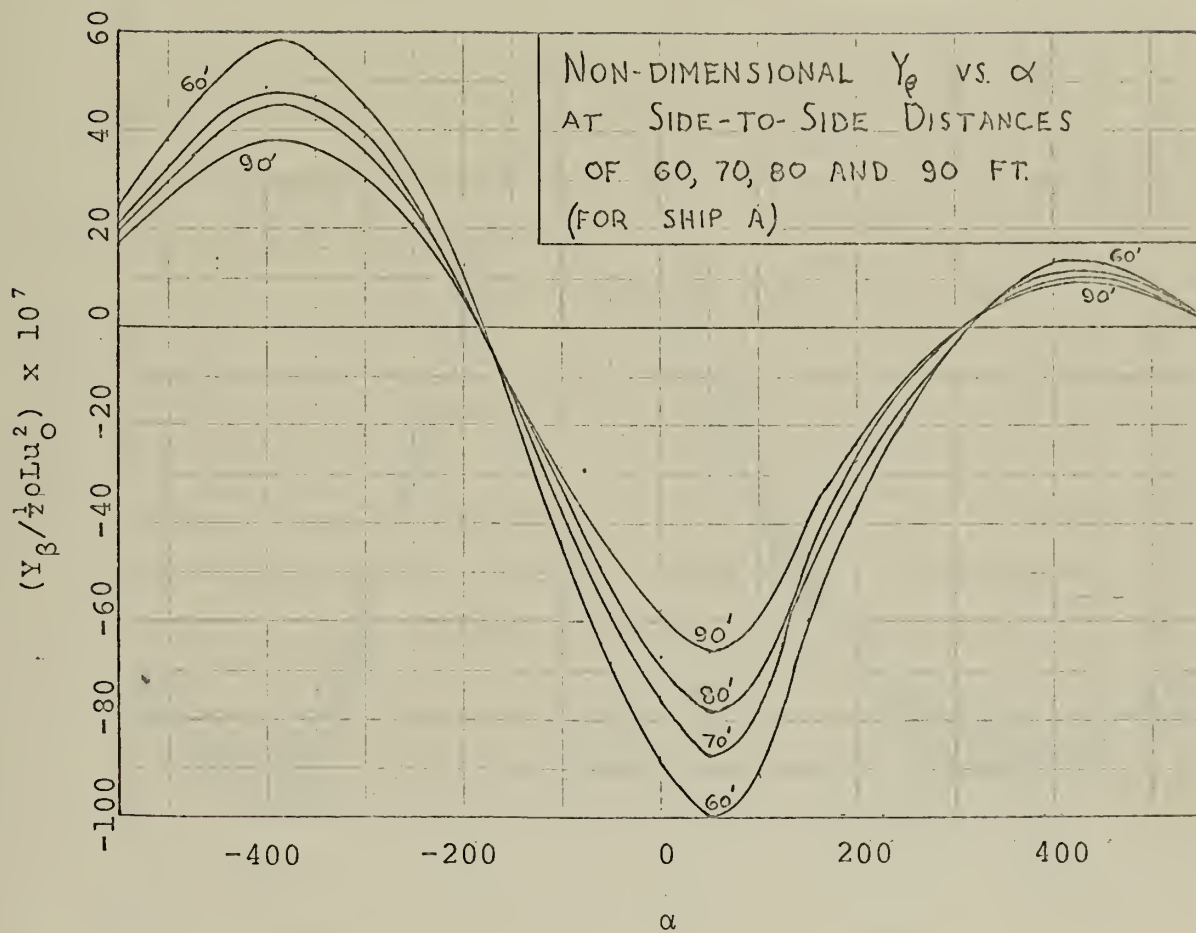
Dimensionless Y_α vs. Longitudinal Separation

FIGURE III-5



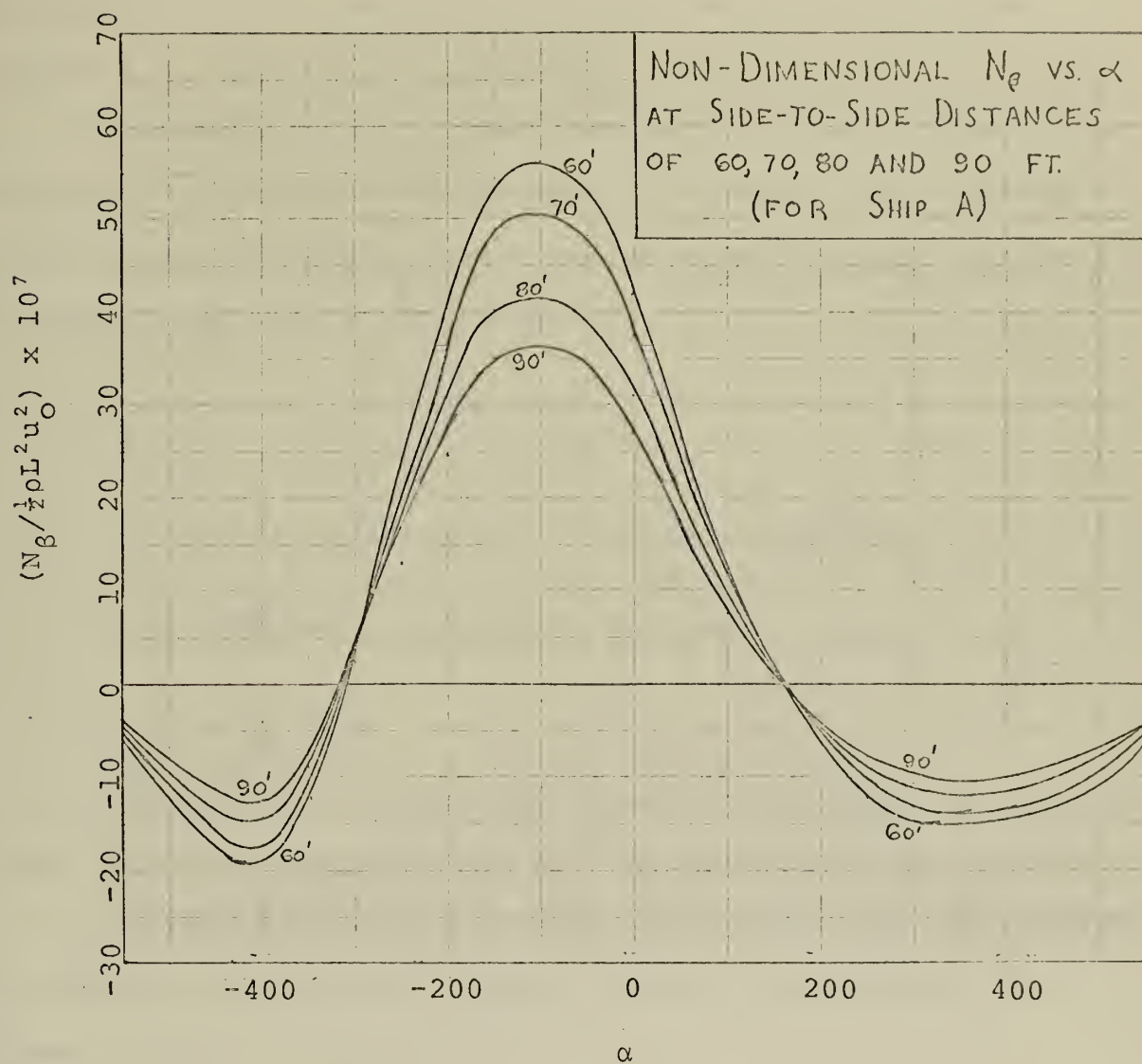
Dimensionless N_α vs. Longitudinal Separation

FIGURE III-6



Dimensionless Y_β vs. Longitudinal Separation

FIGURE III-7



Dimensionless N_β vs. Longitudinal Separation

FIGURE III-8

obtained at a speed of 15 knots, which was also used as the equilibrium speed for the results presented in this thesis. (However, all force, moment and derivative values are presented in non-dimensional form and are valid at other speeds.)

Remembering that Y_u , for example, is $\partial Y / \partial u$ with all other variables at equilibrium values, and knowing Y for a given position at 15 knots, it is a simple matter to get Y at 14 and 16 knots, and then Y_u at 15 knots.

$$a) Y(16) = Y(15) \times (16/15)^2$$

$$b) Y(14) = Y(15) \times (14/15)^2$$

$$c) Y_u(15) = (Y(16) - Y(14)) / ((16 - 14) \times (1.6878)) \quad (3-4)$$

N_u is obtained by a similar process.

C. X_n, Y_n, N_n .

The effects on forces and moments of a change in propeller speed are also required, and are represented by X_n, Y_n , and N_n .

The derivative X_n is determined by using the known values of Effective Horsepower, (EHP), and RPM for a MARINER at speeds around 15 knots.

$$a) X(14\text{kts}) = (EHP(14) \times 550) / (15 \times 1.6878)$$

$$b) X(16) = (EHP(16) \times 550) / (16 \times 1.6878)$$

$$c) X_n(15) = (X(16) - X(14)) / ((RPM(16) - RPM(14)) / 60) \quad (3-5)$$

The value of X_n so obtained is in dimensional form, and is then non-dimensionalized to permit broader applicability.

The single screw MARINER requires a 1.2° port rudder angle to maintain a steady course in open water. [A-4] Since the value of Y_δ for the MARINER is known from reference A-4, it is possible to determine what side force is being generated by this 1.2° rudder angle, which is denoted by $(\delta_o)_{ow}$. When maintaining a straight course in open water, this rudder side force is offsetting the side force caused by the asymmetry of a single screw. Thus, on a steady course, $Y_{rudder} = -Y_{prop}$.

The dimensionless Y_δ given in [A-4] can be converted to its dimensional form, which is in units of lbs/radian. The actual Y force exerted can then be calculated by multiplying the dimensional Y_δ by $(\delta_o)_{ow}/57.3^\circ$, giving a side force in pounds which is the negative of the force being caused by the propeller. The force exerted by the propeller can be expected to vary as the propeller speed squared, and, thus, knowing the propeller side force at a given speed, it is possible to determine how this force varies as propeller speed, n , varies.

$$a) Y_{rudder} = Y_\delta \times NDF(@15 \text{ kts}) \times (\delta_o)_{ow} / (1 \text{ radian})$$

$$b) Y_{prop}(15 \text{ kts}) = -Y_{rudder}(15)$$

$$c) Y_{prop}(14) = Y_{prop}(15) \times (n(14)/n(15))^2$$

$$d) Y_{prop}(16) = Y_{prop}(15) \times (n(16)/n(15))^2$$

$$e) Y_n(15) = (Y_{prop}(16) - Y_{prop}(14)) / (n(16) - n(14))$$

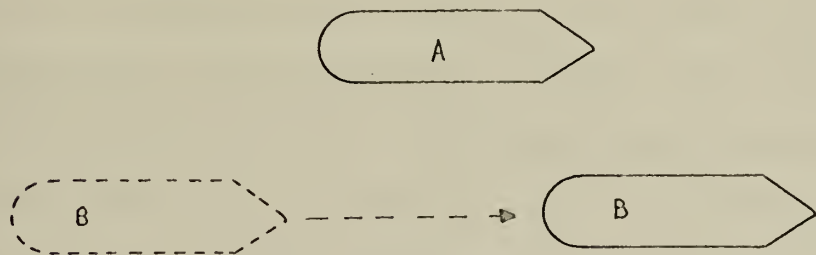
(Where NDF is the non-dimensionalizing factor for Y_δ)

The value of N_n is similarly obtained, and the Y_n and N_n so gotten are suitably non-dimensionalized.

A computer program, program DETERM, was written to determine the values of Y_u , N_u , X_n , Y_n and N_n . The inputs for the program are the values of Y and N at the positions of interest, gotten from program INTCOF, and the EHP, RPM, speed and $(\delta_o)_{ow}$ values for the ship being examined. The values of Y_δ and N_δ in open water are also inputs. Program DETERM is described in detail in Appendix III.

IV. Calculation of Interaction Derivatives for Ship B.

All discussion concerning interaction hydrodynamic derivatives, so far, has been for the OLNA/MARINER ship which is assumed to be Ship A of the two ship system. Ship A serves as the reference for measuring both longitudinal and lateral separations, α and β , and is assumed to be the overtaken ship, located to the left of Ship B, the overtaking ship.

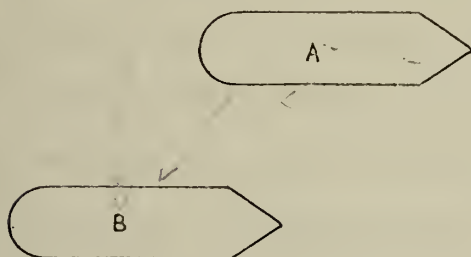


This coincides with the case for the OLNA experiments and all plots of the interaction variables vs. α are for the overtaken ship on the left.

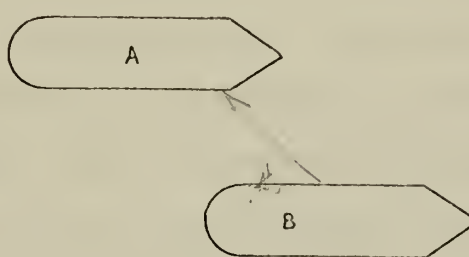
But hydrodynamic derivatives for both ships, at any relative position of interest, are needed to write the equations of motion for the system. Since the modified OLNA/

MARINER used as Ship A is the only one for which it was possible to obtain all required derivatives, Ship B must also be considered to be this type of ship, in order to be able to obtain derivatives for it. If the two ships are identical, the force or moment felt by Ship A at a given α , and β are the negatives of the force or moment felt by Ship B at the same β at $\alpha = -\alpha$. To illustrate:

1.



2.



Y on Ship B at 2 equals $-Y$ on Ship A at 1. If Ship A is attracted to B in position 1, then B will be attracted to A in the same way at 2. But because of the reference system, (as illustrated in Figure II-2), an attraction for Ship A is a positive Y force, while for Ship B an attraction is a negative Y force.

Thus, in program DETERM, the Y and N values entered for Ship B are the negatives of those for Ship A when $\alpha = -\alpha$. As an example, with $STSD = 70'$ and $\alpha = 300'$, the non-dimensional Y for Ship A is $.1652 \times 10^{-3}$. The Y value entered for Ship B in this case is $-.1689 \times 10^{-3}$, which would be the value for Ship A when $\alpha = -300'$. (These values are from the INTCOF

output data of Appendix II.) With the inputs so adjusted, program DETERM will calculate correct values of Y_u and N_u for both ships. (The calculations for X_n , Y_n , and N_n are independent of these considerations). The output of program DETERM is, therefore, X_n , Y_n , and N_n for Ship A, (which are assumed to be the same for Ship B), Y_u and N_u for Ship A and Y_u and N_u for Ship A, if it were in Ship B's position.

To bypass the necessity for considering both ships identical, provision was made to let Ship B be of a length different from Ship A, but of the same form. All form coefficients, plus the length/beam and beam/draft ratios, are assumed the same for both ships. This assumption implies that the non-dimensional values of the hydrodynamic derivatives for Ship B are the same as those for Ship A, (with some sign differences, as in Y_β and N_β), when all derivatives are non-dimensionalized with respect to the length of the ship to which they belong. But for the two ship system, the characteristic length used in the non-dimensionalizing process must be the same for all derivatives. It was decided to use the length between perpendiculars, LBP , of Ship A. Thus, if Ship B is of a length different than that of Ship A, the dimensionless derivatives of Ship A must be multiplied by (LBP_B/LBP_A) to get the dimensionless derivative for B with LBP_A replacing LBP_B in the non-dimensionalizing factor.

This modification of the Ship A derivatives to fit Ship B is done in the early part of a third computer program, SOLVE. (This program is used to actually solve the equations of

motion for the stability roots, as outlined below, but modifies the derivatives, as stated, before solving the equations). SOLVE is described in Appendix IV.

V. The Complete List of the Hydrodynamic Derivatives

Table III-1 lists the derivatives from reference A-4 for the MARINER and these are the values used for the open water derivatives for Ship A, and which are modified to apply to Ship B in program SOLVE.

Table III-2 lists the derivatives associated with propeller speed which were calculated by program DETERM for Ship A. In program SOLVE, they are used for Ship A and modified for Ship B.

The values of Y_{α} , Y_{β} , N_{α} , N_{β} , Y_u and N_u are position-dependent, since the values of Y and N acting as a result of interaction effects vary as the ships change positions relative to each other. In addition, these derivatives, even if ships A and B are identical, will not be the same for the two ships. Table III-3 gives values of these derivatives for the two ships, assuming them identical, for two typical positions of the ships relative to each other.


TABLE III-1

| Derivative | Non-Dimension- alizing Factor | Non-Dimensional Value |
|---|----------------------------------|--------------------------|
| $(X_{\dot{u}}-m)$ | $1/2\rho L^3$ | $-840.\times 10^{-5}$ |
| X_u | $1/2\rho L^2 u_o$ | $-120.\times 10^{-5}$ |
| Y_v | $1/2\rho L^2 u_o$ | -1160.4×10^{-5} |
| $(Y_{\dot{v}}-m)$ | $1/2\rho L^3$ | $-1546.\times 10^{-5}$ |
| $(Y_{\dot{\psi}}-mu_o)$ | $1/2\rho L^3 u_o$ | $-499.\times 10^{-5}$ |
| $(Y_{\dot{\psi}}-mx_g)$ | $1/2\rho L^4$ | 8.6×10^{-5} |
| Y_{δ} | $1/2\rho L^2 u_o^2$ | 277.9×10^{-5} |
| N_v | $1/2\rho L^3 u_o^2$ | -263.5×10^{-5} |
| $(N_{\dot{v}}-mx_g)$ | $1/2\rho L^4$ | 22.7×10^{-5} |
| $(N_{\dot{\psi}}-mx_g u_o)$ | $1/2\rho L^4 u_o$ | $-166.\times 10^{-5}$ |
| $(N_{\dot{\psi}}-I_z)$ | $1/2\rho L^5$ | -82.9×10^{-5} |
| N_{δ} | $1/2\rho L^3 u_o^2$ | -138.8×10^{-5} |
| <p>ρ = mass density of sea water 1.9905 lb-sec²/ft⁴</p> <p>L = LBP Ship A, in ft</p> <p>u_o = equilibrium speed in ft/sec</p> | | |

TABLE III-2

| Derivative | Non-Dim. Factor | Non-Dim. Value |
|------------|-------------------|----------------------|
| X_n | $1/2\rho L^3 u_o$ | 4.62×10^{-5} |
| Y_n | $1/2\rho L^3 u_o$ | $-.52\times 10^{-5}$ |
| N_n | $1/2\rho L^4 u_o$ | $.26\times 10^{-5}$ |

TABLE III-3

| $\alpha=0'$; STSD=60' | | | |
|-----------------------------|---|--------------------------|--------------------------|
| Deriv. | Non-Dim. Factor | Ship A Non-Dim. Value | Ship B Non-Dim. Value |
| Y_u | $1/2\rho L^2 u_o$ | 152.5×10^{-5} | -152.5×10^{-5} |
| N_u | $1/2\rho L^3 u_o$ | -67.1×10^{-5} | 67.1×10^{-5} |
| Y_α | $1/2\rho L u_o^2$ | $.16 \times 10^{-5}$ | $.16 \times 10^{-5}$ |
| Y_β | $1/2\rho L u_o^2$ | $-.87 \times 10^{-5}$ | $.87 \times 10^{-5}$ |
| N_α | $1/2\rho L^2 u_o^2$ | $.126 \times 10^{-5}$ | $.126 \times 10^{-5}$ |
| N_β | $1/2\rho L^2 u_o^2$ | $.42 \times 10^{-5}$ | $-.42 \times 10^{-5}$ |
| $\alpha = -400'$; STSD=80' | | | |
| Y_u | Same | -47.6×10^{-5} | -4.29×10^{-5} |
| N_u | | 17.3×10^{-5} | -16.7×10^{-5} |
| Y_α | as | $.044 \times 10^{-5}$ | $-.093 \times 10^{-5}$ |
| Y_β | Above  | $.447 \times 10^{-5}$ | $-.111 \times 10^{-5}$ |
| N_α | | $-.068 \times 10^{-5}$ | $-.035 \times 10^{-5}$ |
| N_β | | $-.14 \times 10^{-5}$ | $.098 \times 10^{-5}$ |

VI. Method of Solving for Stability Roots

In Chapter II, it was shown that the stability characteristic equation for the system was obtained by evaluating the determinant formed by the coefficients of the variables. These coefficients are polynomials in D , and it was stated that the operator could be treated as an algebraic quantity,

so that the result of the determinant evaluation was another polynomial in Ω , whose roots were the stability roots.

The coefficient determinant to be evaluated is a 6 x 6 determinant, given as equation (2-17). The determinant is evaluated by pivotal condensation ([A-13], pg. 155, problem 4-21). An illustration of pivotal condensation follows:

$$\begin{aligned}
 & \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} = a_{11} \times \begin{bmatrix} 1 & \frac{a_{12}}{a_{11}} & \dots & \frac{a_{1n}}{a_{11}} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \\
 & = a_{11} \times \begin{bmatrix} 1 & 0 & \dots & 0 \\ a_{21} & a'_{22} & \dots & a'_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a'_{n2} & \dots & a'_{nn} \end{bmatrix} = a_{11} \times \begin{bmatrix} a'_{22} & \dots & a'_{2n} \\ \vdots & & \vdots \\ a'_{n2} & \dots & a'_{nn} \end{bmatrix} \quad (3-7)
 \end{aligned}$$

Where $a'_{ij} = a_{ij} - (a_{1i}/a_{11})(a_{1j})$

The procedure would then be repeated with $\begin{bmatrix} a'_{22} & \dots & a'_{2n} \\ \vdots & & \vdots \\ a'_{n2} & \dots & a'_{nn} \end{bmatrix}$

The matrix properties whereby the interchanging of two rows or two columns changes the sign of the determinant are also utilized. After three applications of pivotal condensation and use of these two matrix properties, the determinant

in (2-17) becomes:

$$(AX1) (BX3') (BN6) \times \begin{vmatrix} AY4 & AY5 & AY2 \\ BY4' & BY5' & BY2' \\ AN4 & AN5 & AN2 \end{vmatrix} \quad (3-8)$$

Where: $BX3' = BX3 - BX1(AX3/AX1)$
 $BY4' = BY4 - BY6(BN4/BN6)$
 $BY5' = BY5 - BY6(BN4/BN6)$
 $BY2' = BY2 - BY6(BN4/BN6)$ (3-9)

Carrying the evaluation of (3-8) to its conclusion, and utilizing the relationships of (3-9), the determinant becomes:

$$\begin{aligned} DET = & [(AX1) (BX3) - (BX1) (AX3)] \\ & \times \{ [(BN6) (BY5) - (BY6) (BN5)] \times [(AY4) (AN2) - (AN4) (AY2)] \\ & + [(BN6) (BY2) - (BY6) (BN2)] \times [(AY5) (AN4) - (AN5) (AY4)] \\ & + [(BN6) (BY4) - (BY6) (BN4)] \times [(AY2) (AN5) - (AN2) (AY5)] \} \end{aligned} \quad (3-10)$$

Where the various terms are defined in Table III-4.

Program SOLVE, after appropriately modifying all hydrodynamic derivatives to apply to Ship B as well as Ship A, then calls a subroutine, DETER, to carry out the mathematical operations to evaluate (3-10) at one or more positions and with one or more sets of control system constants for the two ships at the various positions. Each of the terms in (3-10), $AX2$, $BN6$, $AY2$, etc., is a polynomial in θ , and the various multiplications, subtractions and additions are carried out in subroutine DETER by using IBM provided Scientific Subroutine

$$AX1 = X_{n_A}^{k_5} + X_{n_A}^{(k_6^{-k_5} \Delta t_2)} \mathcal{D} - X_{n_A}^{k_6 \Delta t_2} \mathcal{D}^2$$

$$AX3 = X_{u_A} + (X_{\dot{u}_A}^{-m_A}) \mathcal{D}$$

$$AY2 = (Y_{\beta_A} + Y_{\delta_A}^{k_3}) + Y_{\delta_A}^{(k_4^{-k_3} \Delta t_1)} \mathcal{D} - Y_{\delta_A}^{k_4 \Delta t_1} \mathcal{D}^2$$

$$AY4 = Y_{v_A} + (Y_{\dot{v}_A}^{-m_A}) \mathcal{D}$$

$$AY5 = Y_{\delta_A}^{k_1} + ((Y_{\psi_A}^{-m_A} u_O) + Y_{\delta_A}^{(k_2^{-k_1} \Delta t_1)}) \mathcal{D} + ((Y_{\psi_A}^{-m_A} x_g) - Y_{\delta_A}^{k_2 \Delta t_1}) \mathcal{D}^2$$

$$AN2 = (N_{\beta_A} + N_{\delta_A}^{k_3}) + N_{\delta_A}^{(k_4^{-k_3} \Delta t_1)} \mathcal{D} - N_{\delta_A}^{k_4 \Delta t_1} \mathcal{D}^2$$

$$AN4 = N_{v_A} + (N_{\dot{v}_A}^{-m_A} x_g) \mathcal{D}$$

$$AN5 = N_{\delta_A}^{k_1} + ((N_{\psi_A}^{-m_A} x_{u_O}) + N_{\delta_A}^{(k_2^{-k_1} \Delta t_1)}) \mathcal{D} + ((N_{\psi_A}^{-m_A} z_A) - N_{\delta_A}^{k_2 \Delta t_1}) \mathcal{D}^2$$

$$BX1 = X_{n_B}^{k_5} + (X_{n_B}^{(k_6^{-k_5} \Delta t_2)} + X_{u_B}) \mathcal{D} + ((X_{\dot{u}_B}^{-m_B}) - X_{n_B}^{k_6 \Delta t_2}) \mathcal{D}^2$$

$$BX3 = X_{u_B} + (X_{\dot{u}_B}^{-m_B}) \mathcal{D}$$

$$BY2 = (Y_{\beta_B} + Y_{\delta_B}^{k_3}) + (Y_{\delta_B}^{(k_4^{-k_3} \Delta t_1)} + Y_{v_B}) \mathcal{D} + ((Y_{\dot{v}_B}^{-m_B}) - Y_{\delta_B}^{k_4 \Delta t_1}) \mathcal{D}^2$$

$$BY4 = Y_{v_B} + (Y_{\dot{v}_B}^{-m_B}) \mathcal{D}$$

$$BY5 = u_O Y_{v_B} + u_O (Y_{\dot{v}_B}^{-m_B}) \mathcal{D}$$

$$\begin{aligned}
BY6 &= (Y_{\delta} k_1 l_B - u_{OY} v_B) + ((Y_{\psi} - m_B u_O) + Y_{\delta} (k_2 - k_1 \Delta t_1) - u_O (Y_{\dot{V}} - m_B x_B) - Y_{\delta} k_2 \Delta t_1) \mathcal{D}^2 \\
BN2 &= (N_{\beta} + N_{\delta} k_3) + (N_{\delta} (k_4 - k_3 \Delta t_1) + N_{\dot{V}}) \mathcal{D} + ((N_{\dot{V}} - m_B x_B) - N_{\delta} k_4 \Delta t_1) \mathcal{D}^2 \\
BN4 &= N_{\dot{V}} + (N_{\dot{V}} - m_B x_B) \mathcal{D} \\
BN5 &= u_{O\dot{V}} + u_O (N_{\dot{V}} - m_B x_B) \mathcal{D} \\
BN6 &= (N_{\delta} k_1 l_B - u_{O\dot{V}} v_B) + ((N_{\psi} - m_B x_{u_O}) + N_{\delta} (k_2 - k_1 \Delta t_1) - u_O (N_{\dot{V}} - m_B x_B)) \mathcal{D} \\
&\quad + ((N_{\psi} - I_{z_B}) - N_{\delta} k_2 \Delta t_1) \mathcal{D}^2
\end{aligned}$$

Definition of Terms in Determinant

TABLE III-4

Package (SSP) subroutines for arithmetic operations with polynomials. Program SOLVE, (and all the subroutines it calls), is described in detail in Appendix IV.

VII. Control Constant Magnitude

Reference A-11 discusses automatic ship control and utilizes some typical values for the control constants in a system sensitive to ψ and $\dot{\psi}$. Sensitivity to an error parameter, such as ψ , is proportional control, and to an error parameter rate, such as $\dot{\psi}$, is rate control. The equation for rudder control in such a system, (not considering signs), is:

$$\delta = k_1 \psi + k_2 \dot{\psi} \quad (3-11)$$

In [A-11], Schiff and Gimprich use values of k_1 between 1.0 and 5.0, as typical values. The only values used for k_2 are 0 or 1.0.

With $k_1 = 1.0$, a 1° heading error will give rise to 1° of correcting rudder, while with $k_1 = 5.0$, a 1° error will cause a 5° rudder correction. If $k_2 = 1.0$, a heading error rate of $1^\circ/\text{sec.}$ gives a rudder angle of 1° for correction. Thus, k_1 was considered to vary from 1.0 to 5.0, (with smaller values investigated to determine some trends), and k_2 was varied between 0.5 and 4. (Defining δ and ψ to be in radian does not affect k_1 and k_2).

In determining ranges to be used for k_3 and k_4 , which determine sensitivity of the control system to lateral separation error and its rate of change, β and $\dot{\beta}$, no basis

was found in the literature. Thus, these constants were estimated from what the author expected to be reasonable corrections of the rudder angle for lateral separation errors. Rudder angle is sensitive to β as below, (again signs are not considered):

$$\delta = k_3\beta + k_4\dot{\beta} \quad (3-12)$$

Based on the author's experience in underway replenishment operations, it was assumed that an error of 10 feet in β would prompt a rudder correction of about 1° . And a change in $\dot{\beta}$ of 1 ft/sec. was felt to call for 3° of rudder correction.

Therefore:

$$\begin{aligned} k_3 &\approx \frac{1^\circ}{10\text{ft}} \times \frac{1 \text{ radian}}{57.3^\circ} \approx \frac{1}{57.3} \approx .0017 \\ k_4 &\approx \frac{3^\circ}{1\text{ft/sec}} \times \frac{1 \text{ radian}}{57.3^\circ} \approx \frac{3}{57.3} \approx .052 \end{aligned} \quad (3-13)$$

As a result, k_3 was investigated between .0005 and .01 and k_4 between .01 and .1.

A propeller speed control, sensitive to longitudinal separation error and its rate, α and $\dot{\alpha}$, was assumed in the theory. For this sensitivity, not considering signs:

$$n = k_5\alpha + k_6\dot{\alpha} \quad (3-14)$$

To get an idea of the magnitude of k_5 and k_6 , estimates were resorted to. It was assumed that a typical error of 10 feet in α would prompt an n change of 1 RPM, (about 1/4 knot for a MARINER) and a 1 ft/sec. value for $\dot{\alpha}$ would prompt a change of 4 RPM. Therefore:

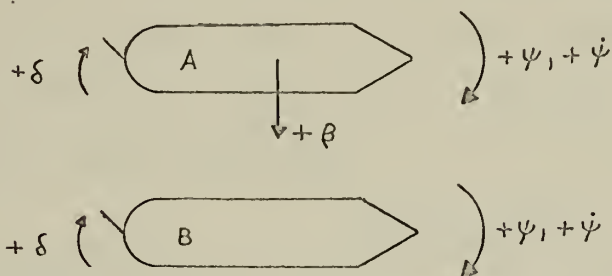
$$k_5 \approx \frac{1 \text{ RPM}}{10 \text{ ft}} \times \frac{1}{60 \text{ RPM/RPS}} \approx \frac{1}{600} \approx .0016$$

$$k_6 \approx \frac{4 \text{ RPM}}{1 \text{ ft/sec}} \times \frac{1}{60 \text{ RPM/RPS}} \approx \frac{1}{15} \approx .066 \quad (3-15)$$

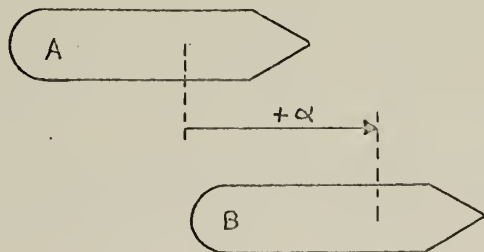
So k_5 was assumed to be between .0005 and .01 in value, and k_6 between .01 and .2.

VIII. Control Constant Signs

When either ship develops a positive error in ψ or $\dot{\psi}$, a positive rudder angle is required to correct for the error. Thus k_1 and k_2 are positive for both ships.



As β or $\dot{\beta}$ increase in a positive direction, it is desired to turn Ship B to the left and Ship A to the right, to compensate. Thus as β increases, a negative δ is required on Ship A, but a positive δ is wanted for Ship B. Therefore k_3 and k_4 are negative for Ship A and positive for Ship B.



As α increases (or $-\alpha$ decreases in magnitude), it is desired to increase Ship A's speed and decrease Ship B's. Therefore k_5 and k_6 are positive for Ship A and negative for Ship B.

Both rudder and propeller response will have time lags associated with them. The rudder, however, can be expected to respond more quickly than propeller speed. Thus rudder time lags from 0 to 4 seconds, and propeller time lags from 0 to 6 seconds are examined. (The desire is to determine the way in which time lags affect stability, and errors in the actual values used will not affect trends developed.)

To summarize, Table III-5 lists all the control system parameter ranges examined.

TABLE III-5

| Ship A | Ship B |
|-----------------------------|-----------------------------|
| $1. \leq k_1 \leq 5.$ | $1. \leq k_1 \leq 5.$ |
| $.5 \leq k_2 \leq 4.$ | $.5 \leq k_2 \leq 4.$ |
| $-.01 \leq k_3 \leq -.0005$ | $.0005 \leq k_3 \leq .01$ |
| $-.1 \leq k_4 \leq -.01$ | $.01 \leq k_4 \leq .1$ |
| $.0005 \leq k_5 \leq .01$ | $-.01 \leq k_5 \leq -.0005$ |
| $.01 \leq k_6 \leq .2$ | $-.2 \leq k_6 \leq -.01$ |
| $0 \leq \Delta t_1 \leq 4.$ | $0 \leq \Delta t_1 \leq 4.$ |
| $0 \leq \Delta t_2 \leq 6.$ | $0 \leq \Delta t_2 \leq 6.$ |

Chapter IV

RESULTS, CONCLUSIONS AND RECOMMENDATIONS

I. Introduction

The intent of this thesis is to develop and solve the equations of motion for the two ship system, and to obtain the stability roots for it. The method, with the computer programs used to perform the calculations, (which would be prohibitive by hand), permits determination of whether or not the system is stable for a given set of control constants and time lags at given positions. In order to examine the accuracy of the solutions and correctness of the method, various sets of control system constants, for various ship positions, were used as inputs to the computer programs and the effect on the stability of the system observed.

It is beyond the scope of this thesis to attempt an optimization of, or even complete examination of, the effects of variations in the automatic control parameters upon system stability. The purpose is to present the method, with only enough calculation to assess the value of the theoretical formulation and computer solution. As a result, the various combinations of controls and ship positions examined are by no means exhaustive---but are representative.

II. Results

A few sets of control constants were evaluated at each of four positions, for ships of different size, with and without

time lag, and are presented in Appendix IV as typical output from program SOLVE.

To perform a somewhat more complete examination of system stability, a single position, ($\alpha = 0'$, STSD = $90'$), was chosen, and a total of 91 combinations of control constants and time lags was run. The results of these runs are summarized in Tables IV-1, IV-2, and IV-3. In these tables, the number of zero roots, number of negative roots and number of positive roots obtained for the input control constants are indicated. In addition, the largest root for each condition is listed, unless the largest root is zero. In this case the largest negative root is listed. The effects of changes in some of the control constants on the value of the largest root are shown in Figures IV-1 through IV-5.

III. Discussion of Results

The first part of Table IV-1 summarizes the effect of varying k_1 , with all other control parameters zero and no time lags. These results are illustrated in Figure IV-1. Varying k_{1A} , alone, from a very low value to its maximum, decreases instability for a time, (the value of the largest root is positive, but decreasing), but then instability increases as k_{1A} increases. When k_{1A} and k_{1B} are both increased through this same range, the system instability is lessened, with stability actually achieved for part of the range, but the trend for instability to increase for the higher k_1 values is still present.

The system investigated to get the results presented in this chapter consisted of two identical ships. This permitted a check of the method to see that the solution is symmetrical for this symmetrical system. As a check, a run was made with $k_{1B} = 3$ and $k_{1A} = 0$, which yielded results identical to those when $k_{1A} = 3$ and $k_{1B} = 0$. Thus the system of equations, and their solution, appear to be correct in at least this respect.

The second and third parts of Table IV-1 summarize the results of varying k_3 and k_5 for the two ships, with all other constants zero. As can be seen from the table, and from Figure IV-2, neither k_3 nor k_5 , alone, even when present on both ships, is sufficient to bring about system stability. As k_3 is increased, for one or both ships, with all other k 's zero, system instability is actually increased. And as k_5 is increased, the value of the largest root is unaffected, though the values of the other roots do change slightly. (The symmetry of the solution was again demonstrated for these constants.)

Next, k_{3A} , k_{3B} , k_{5A} and k_{5B} were all given values in their expected ranges, while k_{1A} and k_{1B} were varied. The results are summarized in the fourth part of Table IV-1 and illustrated in Figure IV-3. In these circumstances, increasing k_{1A} and k_{1B} reduces system instability until a stable condition is attained.

TABLE IV-1

Summary of Stability Roots Obtained for Various
Combinations of Control System Constants With No Time Lags

$\alpha = 90^\circ$; STSD = 0'; $V_O = 15$ kts.

Ship A LBP = 528.5'; Ship B LBP = 528.5'

All unlisted $k_i = 0$

| 1. | k_{1A} | k_{1B} | # 0 Rts. | # < 0 Rts. | # > 0 Rts. | Largest Rt. $\neq 0$ | Stable? |
|----|----------|----------|-------------|---------------|---------------|-------------------------|---------|
| | 0 | 0 | 2 | 6 | 2 | $.1227 \times 10^{-2}$ | no |
| | .001 | 0 | 1 | 7 | 2 | $.1201 \times 10^{-2}$ | no |
| | .01 | 0 | 1 | 7 | 2 | $.9343 \times 10^{-3}$ | no |
| | .1 | 0 | 1 | 7 | 2 | $.3293 \times 10^{-3}$ | no |
| | 1. | 0 | 1 | 7 | 2 | $.6980 \times 10^{-3}$ | no |
| | 2. | 0 | 1 | 7 | 2 | $.7186 \times 10^{-3}$ | no |
| | 3. | 0 | 1 | 7 | 2 | $.7255 \times 10^{-3}$ | no |
| | 4. | 0 | 1 | 7 | 2 | $.7289 \times 10^{-3}$ | no |
| | 5. | 0 | 1 | 7 | 2 | $.7310 \times 10^{-3}$ | no |
| | 0 | 3. | 1 | 7 | 2 | $.7255 \times 10^{-3}$ | no |
| | .001 | .001 | 1 | 7 | 2 | $.1175 \times 10^{-2}$ | no |
| | .01 | .01 | 1 | 7 | 2 | $.6861 \times 10^{-3}$ | no |
| | .1 | .1 | 1 | 9 | 0 | $-.2123 \times 10^{-2}$ | yes |
| | 1. | 1. | 1 | 9 | 0 | $-.1877 \times 10^{-3}$ | yes |
| | 2. | 2. | 1 | 9 | 0 | $-.9270 \times 10^{-4}$ | yes |
| | 3. | 3. | 1 | 9 | 0 | $-.6111 \times 10^{-4}$ | yes |
| | 4. | 4. | 1 | 9 | 0 | $-.4534 \times 10^{-4}$ | yes |
| | 5. | 5. | 1 | 9 | 0 | $-.3587 \times 10^{-4}$ | yes |

TABLE IV-1
(con't)

| 2. | k_{3A} | k_{3B} | # 0 Rts. | #<0 Rts. | #>0 Rts. | Largest Rt. $\neq 0$ | Stable? |
|----|----------|----------|-------------|-------------|-------------|-------------------------|---------|
| | -.005 | 0 | 2 | 6 | 2 | $.2809 \times 10^{-1}$ | no |
| | -.01 | 0 | 2 | 6 | 2 | $.3720 \times 10^{-1}$ | no |
| | -.03 | 0 | 2 | 6 | 2 | $.5848 \times 10^{-1}$ | no |
| | -.05 | 0 | 2 | 6 | 2 | $.7242 \times 10^{-1}$ | no |
| | -.07 | 0 | 2 | 6 | 2 | $.8348 \times 10^{-1}$ | no |
| | 0 | .01 | 2 | 6 | 2 | $.3720 \times 10^{-1}$ | no |
| | -.005 | .005 | 2 | 6 | 2 | $.3720 \times 10^{-1}$ | no |
| | -.008 | .008 | 2 | 6 | 2 | $.4509 \times 10^{-1}$ | no |
| | -.01 | .01 | 2 | 6 | 2 | $.4940 \times 10^{-1}$ | no |
| | -.03 | .03 | 2 | 6 | 2 | $.7821 \times 10^{-1}$ | no |
| | -.05 | .05 | 2 | 6 | 2 | $.9741 \times 10^{-1}$ | no |
| | -.07 | .07 | 2 | 6 | 2 | .1122 | no |

TABLE IV-1
(con't)

| 3. | k_{5A} | k_{5B} | # 0 Rts. | #<0 Rts. | #>0 Rts. | Largest Rt. $\neq 0$ | Stable? |
|--|----------|----------|-------------|-------------|-------------|-------------------------|---------|
| | .005 | 0 | 1 | 7 | 2 | $.1227 \times 10^{-2}$ | no |
| | .008 | 0 | 1 | 7 | 2 | " | no |
| | .01 | 0 | 1 | 7 | 2 | " | no |
| | .03 | 0 | 1 | 7 | 2 | " | no |
| | .05 | 0 | 1 | 7 | 2 | " | no |
| | .07 | 0 | 1 | 7 | 2 | " | no |
| | 0 | -.01 | 1 | 7 | 2 | " | no |
| | .005 | -.005 | 1 | 7 | 2 | " | no |
| | .008 | -.008 | 1 | 7 | 2 | " | no |
| | .01 | -.01 | 1 | 7 | 2 | " | no |
| | .03 | -.03 | 1 | 7 | 2 | " | no |
| | .05 | -.05 | 1 | 7 | 2 | " | no |
| | .07 | -.07 | 1 | 7 | 2 | " | no |
| $k_{3A} = -.005 = -k_{3B}; k_{5A} = .01 = -k_{5B}$ | | | | | | | |
| 4. | k_{1A} | k_{1B} | # 0 Rts. | #<0 Rts. | #>0 Rts. | Largest Rt. $\neq 0$ | Stable? |
| | 1. | 1. | 0 | 8 | 2 | $.2982 \times 10^{-1}$ | no |
| | 2. | 2. | 0 | 8 | 2 | $.2163 \times 10^{-1}$ | no |
| | 3. | 3. | 0 | 8 | 2 | $.1245 \times 10^{-1}$ | no |
| | 4. | 4. | 0 | 8 | 2 | $.2264 \times 10^{-2}$ | no |
| | 5. | 5. | 0 | 10 | 0 | $-.3403 \times 10^{-2}$ | yes |

TABLE IV-1
(con't)

| $k_{1A} = k_{2B} = 2.; k_{3A} = -.005 = -k_{3B}; k_{5A} = .01 = -k_{5B}$ | | | | | | | |
|--|----------|----------|-------------|-------------|-------------|-------------------------|---------|
| 5. | k_{2A} | k_{2B} | # 0 Rts. | #<0 Rts. | #>0 Rts. | Largest Rt. $\neq 0$ | Stable? |
| | .5 | .5 | 0 | 8 | 2 | $.2126 \times 10^{-1}$ | no |
| | 1. | 1. | 0 | 8 | 2 | $.2090 \times 10^{-1}$ | no |
| | 2. | 2. | 0 | 8 | 2 | $.2018 \times 10^{-1}$ | no |
| | 3. | 3. | 0 | 8 | 2 | $.1950 \times 10^{-1}$ | no |
| | 4. | 4. | 0 | 8 | 2 | $.1883 \times 10^{-1}$ | no |
| | k_{4A} | k_{4B} | | | | | |
| | -.01 | .01 | 0 | 8 | 2 | $.1996 \times 10^{-1}$ | no |
| | -.04 | .04 | 0 | 8 | 2 | $.1595 \times 10^{-1}$ | no |
| | -.07 | .07 | 0 | 8 | 2 | $.1427 \times 10^{-1}$ | no |
| | -.1 | .1 | 0 | 8 | 2 | $.1499 \times 10^{-1}$ | no |
| | k_{6A} | k_{6B} | | | | | |
| | .01 | -.01 | 0 | 8 | 2 | $.2163 \times 10^{-1}$ | no |
| | .05 | -.05 | 0 | 8 | 2 | " | no |
| | .1 | -.1 | 0 | 8 | 2 | " | no |
| | .2 | -.2 | 0 | 8 | 2 | " | no |

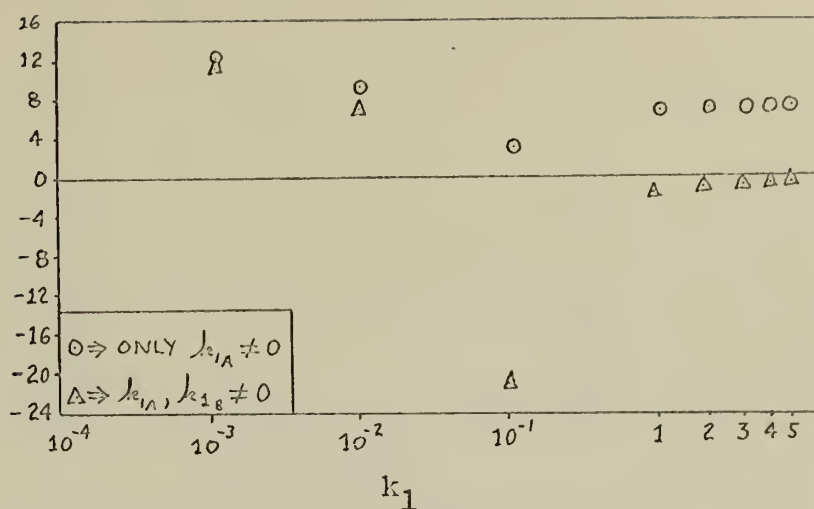
TABLE IV-1
(cont)

$$k_{1A} = k_{1B} = 2.; \quad k_{3A} = -.005 = -k_{3B}; \quad k_{5A} = .01 = -k_{5B};$$

$$k_{2A} = k_{2B}; \quad k_{4A} = -k_{4B}; \quad k_{6A} = -k_{6B}$$

| 6. | k_2 | $ k_4 $ | $ k_6 $ | # 0 Rts. | #<0 Rts. | #>0 Rts. | Largest Rt. $\neq 0$ | Stable? |
|----------------------------------|-------|---------|---------|-------------|-------------|-------------|-------------------------|---------|
| | .5 | .01 | .01 | 0 | 8 | 2 | $.1956 \times 10^{-1}$ | no |
| | 1. | .04 | .05 | 0 | 8 | 2 | $.1493 \times 10^{-1}$ | no |
| | 2. | .07 | .1 | 0 | 8 | 2 | $.1181 \times 10^{-1}$ | no |
| | 4. | .1 | .2 | 0 | 8 | 2 | $.9429 \times 10^{-2}$ | no |
| Same, but $k_{1A} = k_{1B} = 3.$ | | | | | | | | |
| 7. | .5 | .01 | .1 | 0 | 8 | 2 | $.1030 \times 10^{-1}$ | no |
| | 1. | .04 | .1 | 0 | 8 | 2 | $.6420 \times 10^{-2}$ | no |
| | 2. | .05 | .1 | 0 | 8 | 2 | $.4783 \times 10^{-2}$ | no |
| Same, but $k_{1A} = k_{1B} = 4.$ | | | | | | | | |
| 8. | 1. | .04 | .1 | 0 | 10 | 0 | $-.1449 \times 10^{-2}$ | yes |
| | 2. | .04 | .1 | 0 | 10 | 0 | $-.2810 \times 10^{-2}$ | yes |

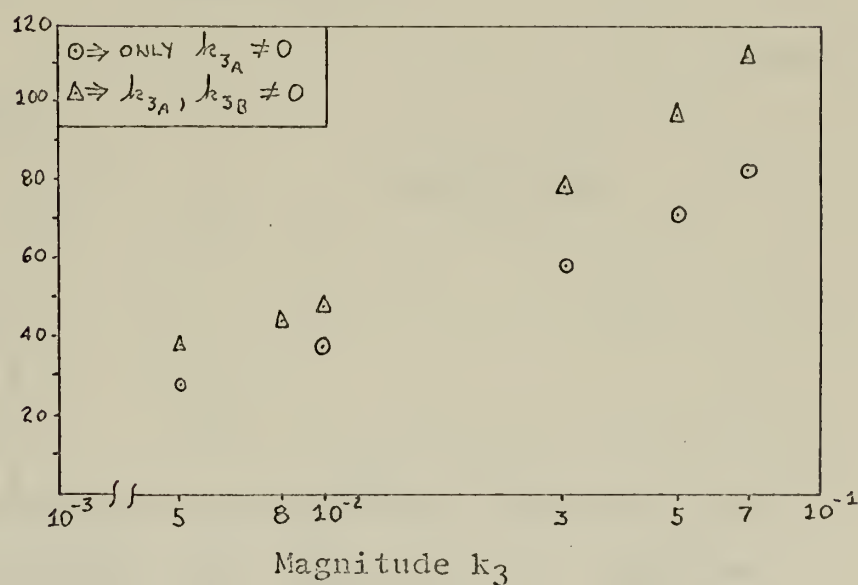
Largest
Root $\times 10^4$



k_1 vs. Largest Root
for all $\Delta t=0$

FIGURE IV-1

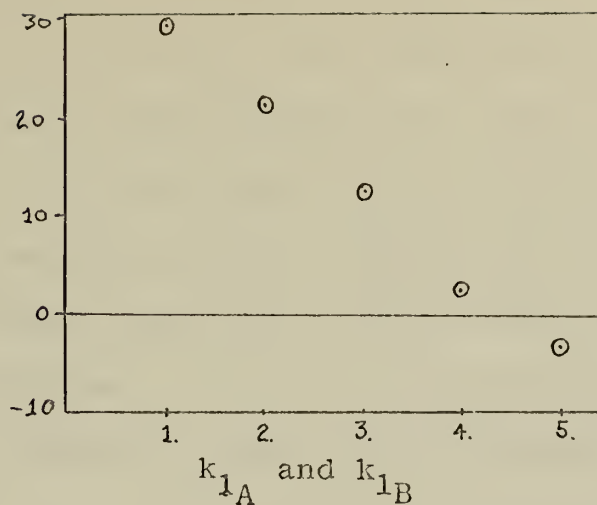
Largest
Root $\times 10^3$



k_3 vs. Largest Root
for all $\Delta t=0$

FIGURE IV-2

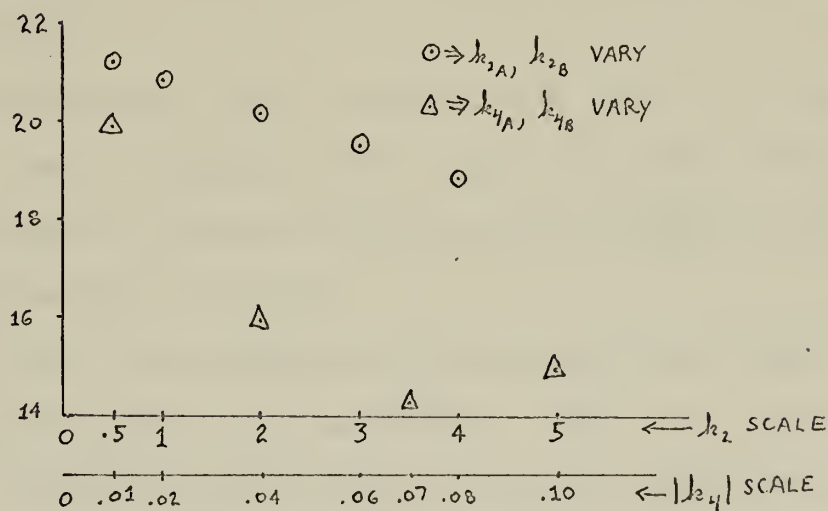
Largest
Root $\times 10^3$



k_{1A} and k_{1B} vs. Largest Root
($k_{3A} = -.005 = -k_{3B}$; $k_{5A} = .01 = -k_{5B}$;
All $\Delta t = 0$)

FIGURE IV-3

Largest
Root $\times 10^3$



k_2 and $|k_4|$ vs Largest Root
($k_{1A} = k_{1B} = 2.$; $k_{3A} = -.005 = -k_{3B}$;
 $k_{5A} = .01 = -k_{5B}$; All $\Delta t = 0$)

FIGURE IV-4

Then, with the proportional constants for both ships, (k_1 , k_3 and k_5), set at values within their ranges, the rate constants, (k_2 , k_4 and k_6), were systematically increased. This step is summarized in Table IV-1, part 5, and the effects for k_2 and k_4 are illustrated in Figure IV-4. As can be seen, increasing k_2 decreased system instability, and increasing k_4 had the same effect until $|k_4|$ reached about .07, after which instability increased. Varying k_6 had no effect on the largest root, though the other roots changed slowly as k_6 increased.

In part 6 of Table IV-1, the effect of increasing all three rate constants, with the proportional constants fixed, is shown to be a decrease in instability. And in parts 7 and 8, the same trend is shown with the k_1 values being set at first 3.0, then 4.0. This last combination is seen to result in a stable system.

Some investigations with non-zero time lags are summarized in Table IV-2, and in Figure IV-5. As can be seen, in all cases investigated an increase in time lags caused the system to become increasingly unstable.

In Table IV-3, the results of giving only one ship an automatic control system are summarized. For this case all $k_{i_B} = 0$. This table indicates that a stable system can be achieved with only one ship automatically controlled, with control constants in the expected ranges. Physically, this would appear to indicate that one ship has been "captured"

TABLE IV-2

Summary of Stability Roots Obtained for Various
Combinations of Control System Constants With Time Lags

$$\alpha = 90'; \text{ STSD} = 0'; V_0 = 15 \text{ kts.}$$

$$\text{Ship A LBP} = 528.5'; \text{ Ship B LBP} = 528.5'$$

All unlisted $k_i = 0$

$$k_{1A} = k_{1B} = 2.; k_{3A} = -.005 = -k_{3B}; k_{5A} = .01 = -k_{5B}$$

| Δt_{1A} | Δt_{2A} | Δt_{1B} | Δt_{2B} | # 0 Rts. | #<0 Rts. | #>0 Rts. | Largest Rt. $\neq 0$ | Stable? |
|-----------------|-----------------|-----------------|-----------------|-------------|-------------|-------------|-------------------------|---------|
| 1 | 2 | 1 | 2 | 0 | 8 | 2 | $.2399 \times 10^{-1}$ | no |
| 2 | 4 | 2 | 4 | 0 | 6 | 4 | $.2637 \times 10^{-1}$ | no |
| 4 | 6 | 4 | 6 | 0 | 6 | 4 | $.3116 \times 10^{-1}$ | no |

Same, plus $k_{2A} = k_{2B} = 2.$

| | | | | | | | | |
|---|---|---|---|---|---|---|------------------------|----|
| 1 | 2 | 1 | 2 | 0 | 8 | 2 | $.2246 \times 10^{-1}$ | no |
| 2 | 4 | 2 | 4 | 0 | 6 | 4 | $.2477 \times 10^{-1}$ | no |
| 4 | 6 | 4 | 6 | 0 | 6 | 4 | $.2951 \times 10^{-1}$ | no |

Same, plus $k_{4A} = -.04 = -k_{4B}$

| | | | | | | | | |
|---|---|---|---|---|---|---|------------------------|----|
| 1 | 2 | 1 | 2 | 0 | 8 | 2 | $.2027 \times 10^{-1}$ | no |
| 2 | 4 | 2 | 4 | 0 | 6 | 4 | $.2452 \times 10^{-1}$ | no |
| 4 | 6 | 4 | 6 | 0 | 6 | 4 | $.3278 \times 10^{-1}$ | no |

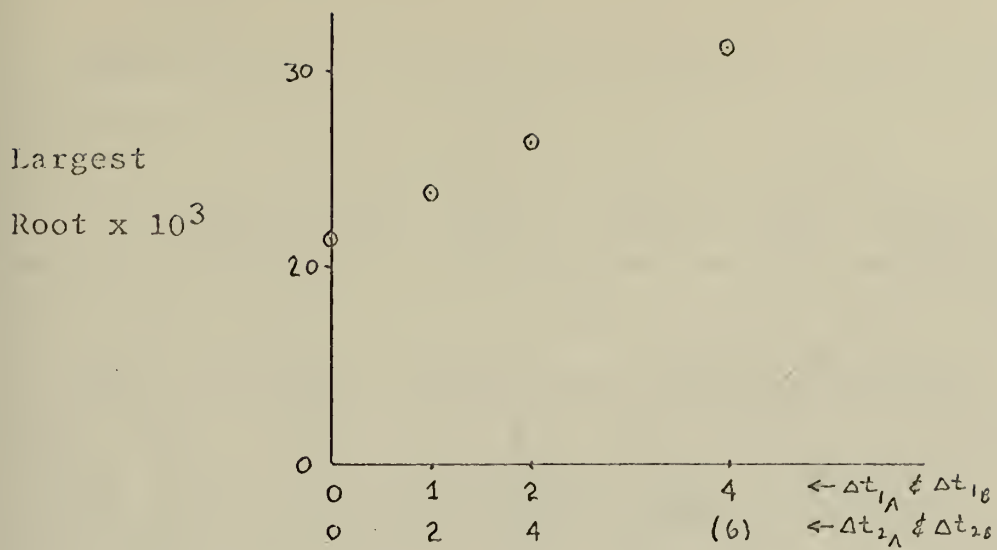
TABLE IV-2
(con't)

Same, plus $k_{6A} = .1 = -k_{6B}$

| Δt_{1A} | Δt_{2A} | Δt_{1B} | Δt_{2B} | # 0 Rts. | #<0 Rts. | #>0 Rts. | Largest Rt. $\neq 0$ | Stable? |
|-----------------|-----------------|-----------------|-----------------|-------------|-------------|-------------|-------------------------|---------|
| 1 | 2 | 1 | 2 | 0 | 8 | 2 | $.2399 \times 10^{-1}$ | no |
| 2 | 4 | 2 | 4 | 0 | 8 | 2 | $.2637 \times 10^{-1}$ | no |
| 4 | 6 | 4 | 6 | 0 | 8 | 2 | $.3116 \times 10^{-1}$ | no |

Same, plus $k_{2A} = k_{2B} = 2.$; $k_{4A} = -.04 = -k_{4B}$; $k_{6A} = .1 = -k_{6B}$

| | | | | | | | | |
|---|---|---|---|---|---|---|------------------------|----|
| 1 | 2 | 1 | 2 | 0 | 8 | 2 | $.1780 \times 10^{-1}$ | no |
| 2 | 4 | 2 | 4 | 0 | 8 | 2 | $.2362 \times 10^{-1}$ | no |
| 4 | 6 | 4 | 6 | 0 | 8 | 2 | $.3473 \times 10^{-1}$ | no |



Time Lags vs. Largest Root

$(k_{1A} = k_{1B} = 2.; k_{3A} = -.005 = -k_{3B};$
 $k_{5A} = .01 = -k_{5B})$

FIGURE IV-5

TABLE IV-3

| | | | | | | | | |
|--|----------|----------|----------|-------------|-------------|-------------|-------------------------|---------|
| Summary of Stability Roots Obtained for Various Combinations of Single Ship Control With No Time Lags | | | | | | | | |
| $\alpha = 90'$; STSD = $0'$; $V_O = 15$ kts. Ship A LBP = $528.5'$; Ship B LBP = $528.5'$ | | | | | | | | |
| $k_{1A} = 2.$; $k_{3A} = -.005$; $k_{5A} = .01$; All $k_{iB} = 0$ | | | | | | | | |
| | k_{2A} | k_{4A} | k_{6A} | # 0 Rts. | #<0 Rts. | #>0 Rts. | Largest Rt. $\neq 0$ | Stable? |
| | .5 | -.01 | .01 | 0 | 8 | 2 | $.6793 \times 10^{-2}$ | no |
| | 1. | -.04 | .05 | 0 | 8 | 2 | $.3092 \times 10^{-2}$ | no |
| | 2. | -.07 | .1 | 0 | 8 | 2 | $.3914 \times 10^{-5}$ | no |
| | 4. | -.1 | .2 | 0 | 10 | 0 | $-.2852 \times 10^{-3}$ | yes |
| $k_{2A} = 2.$; $k_{3A} = -.005$; $k_{4A} = -.05$; $k_{5A} = .01$; $k_{6A} = .1$; $k_{iB} = 0$ | | | | | | | | |
| | k_{1A} | | | | | | | |
| | 3. | | | 0 | 10 | 0 | $-.4373 \times 10^{-3}$ | yes |
| | 4. | | | 0 | 10 | 0 | $-.6003 \times 10^{-3}$ | yes |

by the other through the interaction force and moment. This was an unexpected result and it was felt that it might be indicative of an error in the formulation or solution of the equations.

To investigate this possibility, some extra computer runs, for special circumstances, were made. In these runs, all the interaction force, moment and derivative inputs for program SOLVE were set equal to zero. This is equivalent to the ships being infinitely far apart, where no interaction effects can be felt. In this circumstance, it should prove possible to obtain a stable system with reasonable control constant values for each ship, and, this, in fact, was the case.

A further check was made, where the ships were given control systems with different constants---then run again with the same two sets of control constants, but switched from one ship to the other---both runs made with the interaction effects all set at zero. These two runs should result in identical sets of stability roots if the solution is correctly done, and, did, in fact, do so.

The last check run to verify the accuracy of the solution method was with all control constants zero for both ships. The interaction effects were all left at zero, as well. This, in effect, puts the ships infinitely far apart, with no automatic control, and should result in neutral directional stability. The result of this run was four zero roots and

six negative roots, indicating a dynamically stable system, neutrally stable, probably with regard to heading

IV. Conclusions

The investigation reported herein has prompted the following conclusions:

- A. The methods of formulation and solution of the equations of motion appear to be correct.
- B. It appears possible to achieve a stable system of two ships on parallel courses in close proximity, using automatic control.
- C. System stability is most affected by control sensitivity to heading angle error, ---that is, k_1 terms; and least affected by sensitivity to longitudinal position error and error rate, and ---that is, k_5 and k_6 terms.
- D. It is possible to achieve a stable system with proportional control only, ($k_2 \quad k_4 \quad k_6 \quad 0$).
- E. It is possible, in isolated circumstances, at least, to achieve a stable system with only one ship employing automatic control.
- F. Under some circumstances, increasing the magnitude of a control constant, and thereby increasing system sensitivity to one of the error parameters, can have an adverse effect on system stability, (as illustrated in Figure IV-2).

V. Recommendations

The following recommendations are made for future investigation into the practicality of employing automatic controls for ships engaged in operations alongside and close aboard, as in underway replenishment:

- A. A detailed examination of the theoretical development used in this thesis should be conducted by someone other than the author, to remove any reservation as to the accuracy of the method.
- B. The examination of system stability should be made for two ships whose full set of hydrodynamic derivatives are known. (This would eliminate the need to synthesize the characteristics of two different ships, as the author did with the OLNA/MARINER modification).
- C. The results of any such theoretical investigation should be confirmed by model tests.
- D. The method should be extended to examine the effect of including an outside excitation on the system, such as waves, wind, etc.

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Appendix I

COMPUTER UTILIZATION

- I. Discussion
- II. Flow Chart


Appendix I

COMPUTER UTILIZATION

I. Discussion

Three computer programs have been written as part of this investigation. The three programs, described in Appendices II, III and IV, are used in sequence if the investigation of the stability of a two ship system, such as that studied herein, starts at the level of known information with which this thesis was begun. If more information concerning the interaction of the two ships being studied is available, it may be possible to enter the sequence at a later stage. (This sequence of the programs is illustrated in the overall flow chart, Figure AI-1). Precisely because any possible future use of these programs may commence at a different stage, the three programs were not unified into a single large program. The use of the programs will be summarized in the reverse order from that in which they were used in this work.

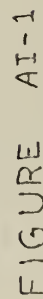
Program SOLVE, the third program used, is the heart of the computer solution and actually solves for the system stability roots. It also requires the greatest amount of known information. If all the interaction and open water hydrodynamic derivatives for Ship A, plus the interaction derivatives for Ship A in Ship B's position, are known, program SOLVE may be used directly.

 Program DETERM can be used to get the force and moment derivatives with respect to propeller speed, n , and the

interaction derivatives Y_u and N_u for both ships, if these quantities are not known. To do this, DETERM uses the values of Y and N at the positions of interest, plus Y_δ , N_δ and δ_0 open water for Ship A as inputs. EHP and RPM data for Ship A are also required inputs. The output of DETERM then forms part of the required input for SOLVE.

Program INTCOF is used to determine the interaction derivatives of Y and N with respect to α and β , and the values of these quantities at the positions being examined can then be used as part of the SOLVE input. In the process of getting these interaction derivatives throughout the range of interest, program INTCOF also calculates Y and N everywhere in that range, using the interpolation technique described in Chapter III and illustrated in Figure III-1.

The way in which INTCOF and DETERM were written was largely dictated by what information was available concerning the interaction phenomena and were steps toward getting all the derivatives needed for SOLVE. As a result, INTCOF, especially, would hardly be likely to be useful to an investigator starting with a different level of known interaction information. (If, for instance, Y and N were known from model tests everywhere in the range of interest, the interpolating portion of INTCOF would not be needed and the derivatives with respect to α and β could be obtained more directly). Program SOLVE, however, should be directly useful for two ship system stability investigations for any two ships whose hydrodynamic derivatives are all known.



Appendix II

PROGRAM INTCOF

- I. Discussion
- II. Flow Chart
- III. Program Listing
- IV. Results

Appendix II

PROGRAM INTCOF

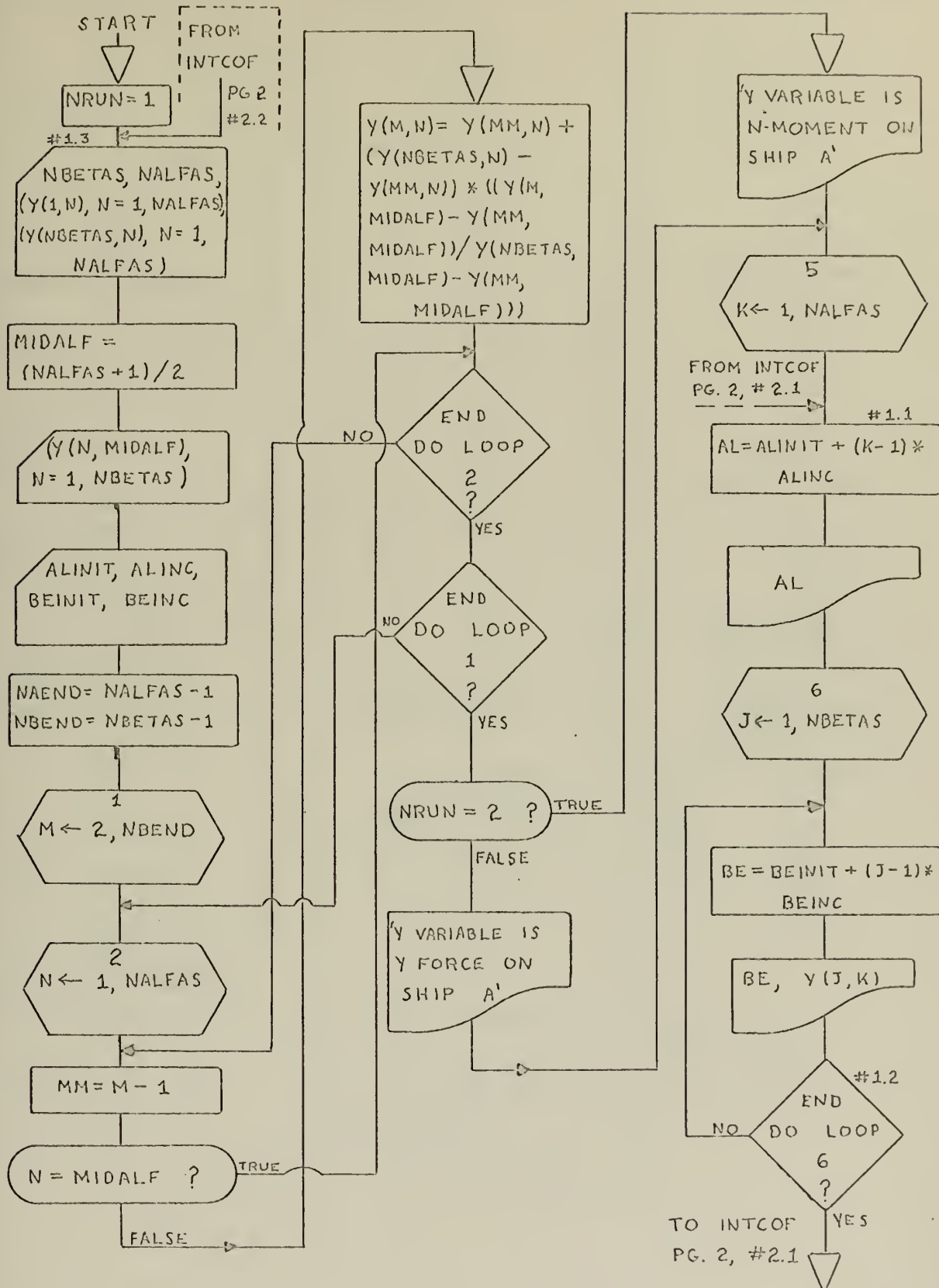
I. Discussion

Program INTCOF takes the values of Y and N where known from reference A-1, interpolates to get Y and N everywhere in the range of interest, then calculates Y_α , Y_β , N_α and N_β everywhere in this range, using subroutines DERIV and DERIVB.

Program INTCOF is highly specialized to the use of the data of reference A-1. Any investigation using Y and N information from another source to get the above four interaction derivatives would be unable to use INTCOF. And, of course, an investigation using the [A-1] data could use the results herein, directly. For these reasons, the input format required for INTCOF will not be described in detail, and the remainder of this Appendix will merely contain a flow chart, program listing and results for INTCOF.

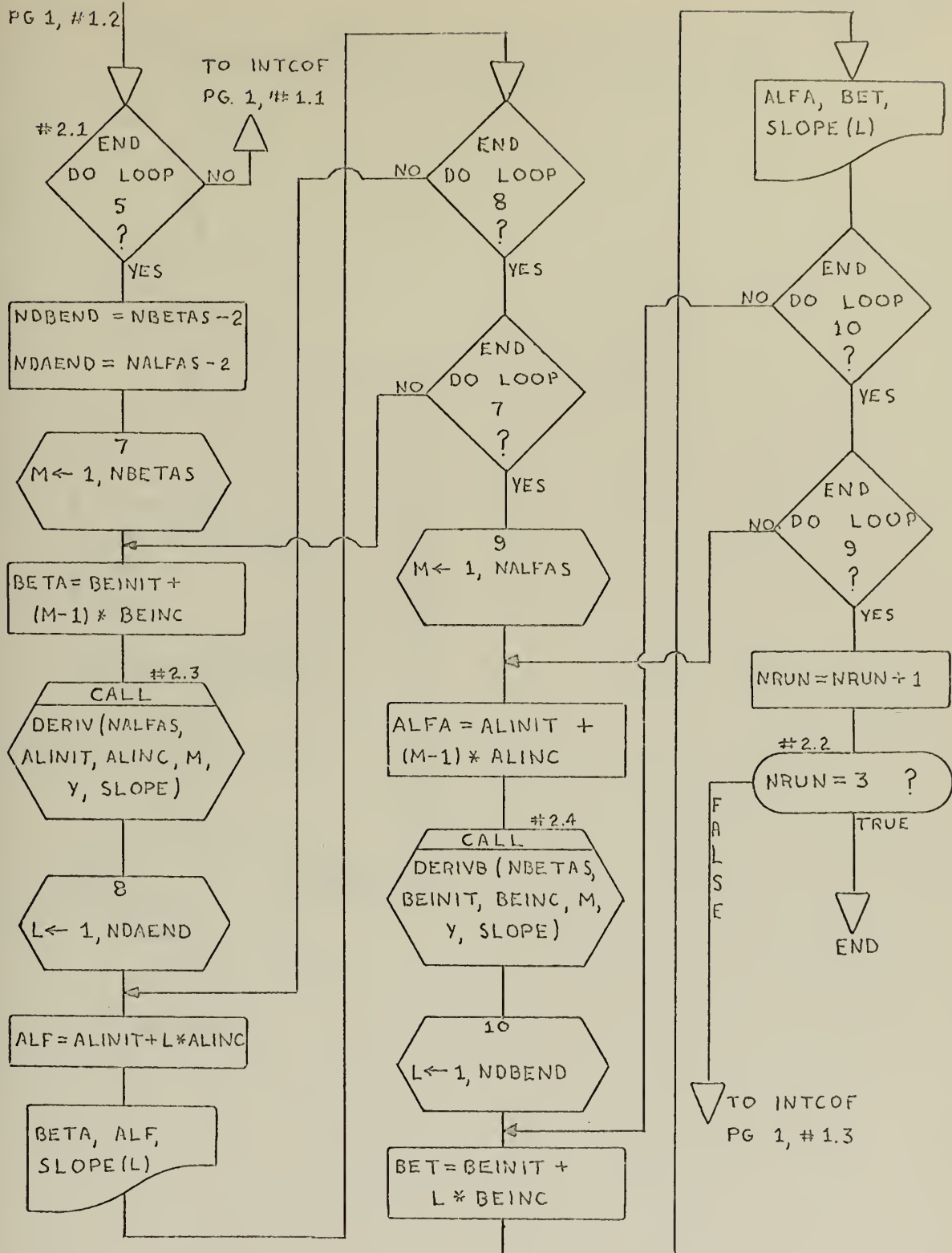
II FLOW CHART — PROGRAM INTCOF

INTCOF PG. 1



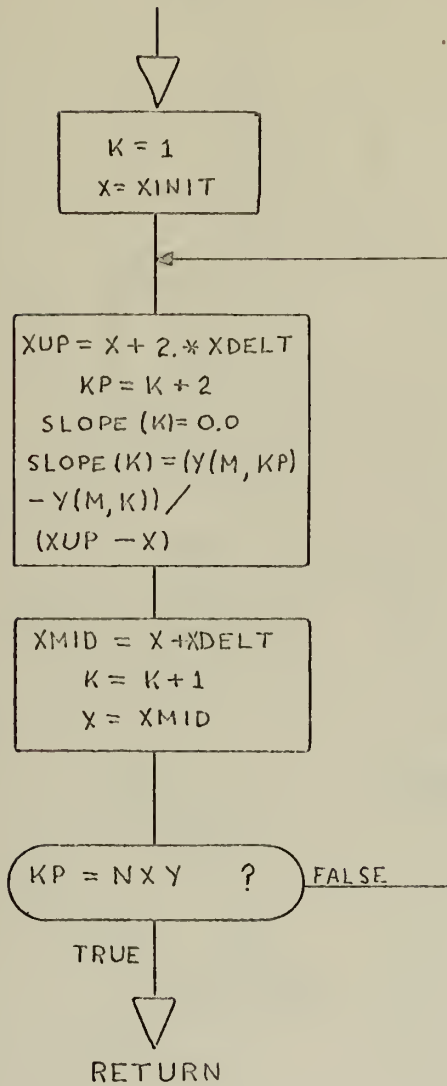
FROM INTCOF
PG 1, #1.2

INTCOF PG. 2

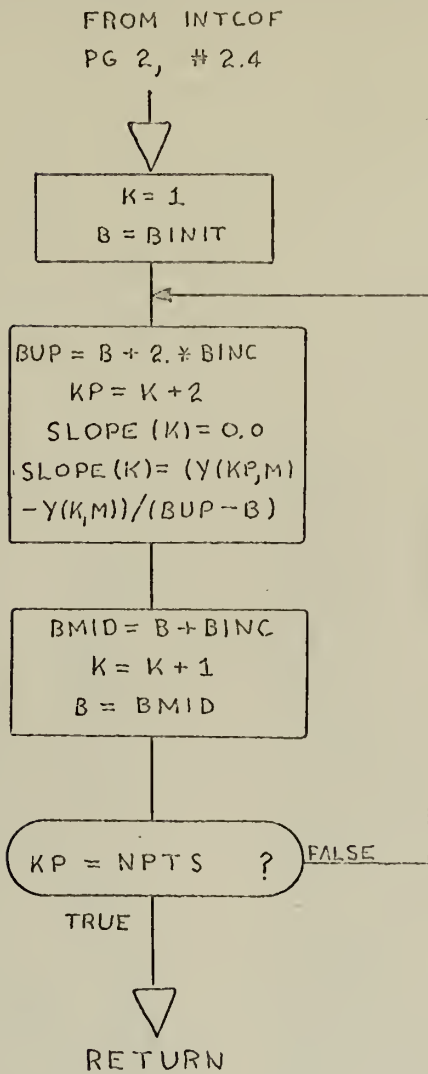


SUBROUTINE DERIV (NXY, XINIT, XDELT, M, Y, SLOPE)

FROM INTCOF
PG. 2, # 2.3



SUBROUTINE DERIVB (NPTS, BINIT, BINCM, Y, SLOPE)



III. INTCOF PROGRAM LISTING

```

C
C
C PROGRAM .....INTCOF.....INTERACTION COEFFICIENTS
C THIS PROGRAM DETERMINES THE VALUES OF INTERACTION
C FORCES AND MOMENTS EVERYWHERE FROM ALFA=-600 TO +600 FT.
C AT 50 FOOT INTERVALS, AND SIDE TO SIDE SEPARATION = 50 TO
C 100 FEET, AT 10 FOOT INTERVALS
C SUBROUTINES ARE THEN CALLED TO DETERMINE DY/D(ALFA),
C DY/D(BETA), DN/D(ALFA) AND DN/D(BETA) AT THESE LOCATIONS
C DIMENSION Y(10,30),SLOPE(30)
100 FORMAT(214)
101 FORMAT(5(E10.2))
102 FORMAT (4F10.2)
200 FORMAT(//,15X,'Y VARIABLE IS Y FORCE ON SHIP A'//
115X,'STSD IS SIDE TO SIDE DISTANCE BETWEEN SHIPS A&R')
201 FORMAT('1',//15X,'Y VARIABLE IS N MOMENT ON SHIP A')
202 FORMAT(15X,'AT ALFA =',F10.2)
203 FORMAT(15X,'STSD= ',F10.1,' Y=',F14.4)
204 FORMAT(15X,'STSD= ',F8.1,' ALFA= ',F8.1,' DY/DA= ',
1,F14.4)
205 FORMAT(15X,'ALFA= ',F8.1,' STSD= ',F8.1,
1,' DY/DB=',F14.4)
206 FORMAT('1',//)

C NRUN IS A COUNTER TO INDICATE THAT THE FIRST TIME THROUGH
C THE Y VARIABLE IS Y FORCE AND THE SECOND TIME THE Y VAR-
C IABLE IS N MOMENT.
NRUN=1

C NBETAS AND NALFAS ARE THE NUMBER OF BETA AND ALFA LO-
C CATIONS USED.
12 READ(5,100)NBETAS,NALFAS

C READ KNOWN Y VARIABLES, FIRST IN ALFA THEN IN BETA DIP-
C FCTION.
READ (5,101) (Y(1,N),N=1,NALFAS)

```



```

      READ (5,101) (Y(NBETAS,N),N=1,NALFAS)
      MIDALF=(NALFAS+1)/2.
      READ(5,101){V(N,MIDALF),N=1,NBETAS}

C      READ INITIAL VALUES AND INCREMENTS FOR ALFA AND BETA
C      READ (5,102) ALINIT, ALINC, REINIT, REINC
      NAEND=NALFAS-1
      NBEND=NBETAS-1

      DO 1 M=2,NBEND
      DO 2 N=1,NALFAS
      MM=M-1
      IF (N.EQ. MIDALF) GO TO 2
      Y(M,N)=Y(MM,N)+(Y(NBETAS,N)-Y(MM,N))*{(Y(N,MIDALF)-Y(M
      1N,MIDALF))/(Y(NBETAS,MIDALF)-Y(MM,MIDALF))}
      2 CONTINUE
      1 CONTINUE
      IF (NRUN.EQ. 2) GO TO 3
      WRITE (6,200)
      GO TO 4
      3 WRITE (6,201)

C      4 DO 5 K=1,NALFAS
      IF(K-7)25,26,37
      27 IF (K-13)25,26,38
      28 IF (K-19)25,26,39
      29 IF (K-25)25,26,25
      26 WRITE (6,206)
      25 AI=ALINIT+(K-1)*ALINC
      WRITE (6,202) AI
      DO 6 J=1,NBETAS
      RE=REINIT+(J-1)*REINC
      WRITE (6,203) RE,V(J,K)
      6 CONTINUE
      5 CONTINUE

```

```

INCE00037
INCE00038
INCE00039
INCE00040
INCE00041
INCE00042
INCE00043
INCE00044
INCE00045
INCE00046
INCE00047
INCE00048
INCE00049
INCE00050
INCE00051
INCE00052
INCE00053
INCE00054
INCE00055
INCE00056
INCE00057
INCE00058
INCE00059
INCE00060
INCE00061
INCE00062
INCE00063
INCE00064
INCE00065
INCE00066
INCE00067
INCE00068
INCE00069
INCE00070
INCE00071
INCE00072

```



```

NDBEND = NRETAS-2
NDAEND = NALFAS-2
WRITE (6,206)

```

```

DO 7 M=1,NRETAS
  IF(M-3)27,28,40
  IF(M-5)27,28,27
28 WRITE(6,206)
27 BETA=BFINIT+(M-1)*REINC
  CALL DERIV(NALFAS,ALINIT,ALINC,M,Y,SLOPE)
DO 8 L=1,NDAEND
  ALF=ALINIT+L*ALINC
  WRITE (6,204) BETA,ALF,SLOPE(L)
8 CONTINUE
7 CONTINUE
  WRITE(6,206)

```

```

DO 9 M=1,NALFAS
  IF(M-10)20,30,41
  IF(M-20)29,30,29
20 WRITE(6,206)
29 ALFA=ALINIT+(M-1)*ALINC
  CALL DERIV(NRETAS,REINIT,REINC,M,Y,SLOPE)
DO 10 L=1,NDBEND
  RET=REINIT+L*REINC
  WRITE (6,205) ALFA,RET,SLOPE(L)
10 CONTINUE
9 CONTINUE

```

```

NRUN=NRUN+1
IF(NRUN.EQ.3) GO TO 11
GO TO 12
11 WRITE(6,206)
  STOP
  END

```

```

INCE00073
INCE00074
INCE00075
INCE00076
INCE00077
INCE00078
INCE00079
INCE00080
INCE00081
INCE00082
INCE00083
INCE00084
INCE00085
INCE00086
INCE00087
INCE00088
INCE00089
INCE00090
INCE00091
INCE00092
INCE00093
INCE00094
INCE00095
INCE00096
INCE00097
INCE00098
INCE00099
INCE0100
INCE0101
INCE0102
INCE0103
INCE0104
INCE0105
INCE0106
INCE0107

```



```

SUBROUTINE DERIV (NXY,XINIT,XDELT,M,Y,SLOPE)
  DIMENSION Y(10,30),SLOPE(30)
  K=1
  X=XINIT
  1001 XUP=X+2.*XDELT
  KP=K+2
  SLOPE(K)=0.
  SLOPE(K)=(Y(M,KP)-Y(M,K))/(XUP-X)
  XMID=X+XDELT
  K=K+1
  X=XMID
  IF(KP,EO,NXY) GO TO 1002
  GO TO 1001
  1002 RETURN
  END

```

```

DERV0001
DERV0002
DERV0003
DERV0004
DERV0005
DERV0006
DERV0007
DERV0008
DERV0009
DERV0010
DERV0011
DERV0012
DERV0013
DERV0014
DERV0015
DERV0016

```



```

C
SUBROUTINE DERIVB (NPTS,RINIT,RINC,M,Y,SLOPE)
  DIMENSION Y(10,30),SLOPE(30)
  K=1
  B=RINIT
  1004 BUP=B+2.*RINC
  KP=K+2
  SLOPE(K)=0.
  SLOPE(K)=(Y(KP,M)-Y(K,M))/(BUP-B)
  BMID=B+RINC
  K=K+1
  B=BMID
  IF(KP.EQ.NPTS) GO TO 1003
  GO TO 1004
  1003 RETURN
  END

```

```

00V80001
00V80002
00V80003
00V80004
00V80005
00V80006
00V80007
00V80008
00V80009
00V80010
00V80011
00V80012
00V80013
00V80014
00V80015
00V80016

```


IV. RESULTS

Y VARIABLE IS Y FORCE ON SHIP A

STSD IS SIDE TO SIDE DISTANCE BETWEEN SHIPS A&B

AT ALFA = -600.00

| | | | |
|-------|-------|----|-------------|
| STSD= | 50.0 | Y= | -0.1400E-03 |
| STSD= | 60.0 | Y= | -0.1263E-03 |
| STSD= | 70.0 | Y= | -0.1160E-03 |
| STSD= | 80.0 | Y= | -0.1058E-03 |
| STSD= | 90.0 | Y= | -0.9753E-04 |
| STSD= | 100.0 | Y= | -0.9000E-04 |

AT ALFA = -550.00

| | | | |
|-------|-------|----|-------------|
| STSD= | 50.0 | Y= | -0.2400E-03 |
| STSD= | 60.0 | Y= | -0.2112E-03 |
| STSD= | 70.0 | Y= | -0.1897E-03 |
| STSD= | 80.0 | Y= | -0.1681E-03 |
| STSD= | 90.0 | Y= | -0.1508E-03 |
| STSD= | 100.0 | Y= | -0.1350E-03 |

AT ALFA = -500.00

| | | | |
|-------|-------|----|-------------|
| STSD= | 50.0 | Y= | -0.3200E-03 |
| STSD= | 60.0 | Y= | -0.2762E-03 |
| STSD= | 70.0 | Y= | -0.2433E-03 |
| STSD= | 80.0 | Y= | -0.2104E-03 |
| STSD= | 90.0 | Y= | -0.1841E-03 |
| STSD= | 100.0 | Y= | -0.1600E-03 |

AT ALFA = -450.00

| | | | |
|-------|-------|----|-------------|
| STSD= | 50.0 | Y= | -0.3800E-03 |
| STSD= | 60.0 | Y= | -0.3225E-03 |
| STSD= | 70.0 | Y= | -0.2793E-03 |
| STSD= | 80.0 | Y= | -0.2362E-03 |
| STSD= | 90.0 | Y= | -0.2016E-03 |
| STSD= | 100.0 | Y= | -0.1700E-03 |

AT ALFA = -400.00

| | | | |
|-------|-------|----|-------------|
| STSD= | 50.0 | Y= | -0.4000E-03 |
| STSD= | 60.0 | Y= | -0.3337E-03 |
| STSD= | 70.0 | Y= | -0.2840E-03 |
| STSD= | 80.0 | Y= | -0.2342E-03 |
| STSD= | 90.0 | Y= | -0.1945E-03 |
| STSD= | 100.0 | Y= | -0.1580E-03 |

AT ALFA = -350.00

| | | | |
|-------|-------|----|-------------|
| STSD= | 50.0 | Y= | -0.3500E-03 |
| STSD= | 60.0 | Y= | -0.2870E-03 |
| STSD= | 70.0 | Y= | -0.2397E-03 |
| STSD= | 80.0 | Y= | -0.1925E-03 |
| STSD= | 90.0 | Y= | -0.1547E-03 |
| STSD= | 100.0 | Y= | -0.1200E-03 |

| | | | |
|-----------|---------|----|-------------|
| AT ALFA = | -300.00 | | |
| STSD= | 50.0 | Y= | -0.2600E-03 |
| STSD= | 60.0 | Y= | -0.2079E-03 |
| STSD= | 70.0 | Y= | -0.1689E-03 |
| STSD= | 80.0 | Y= | -0.1299E-03 |
| STSD= | 90.0 | Y= | -0.9863E-04 |
| STSD= | 100.0 | Y= | -0.7000E-04 |
| AT ALFA = | -250.00 | | |
| STSD= | 50.0 | Y= | -0.1300E-03 |
| STSD= | 60.0 | Y= | -0.9334E-04 |
| STSD= | 70.0 | Y= | -0.6671E-04 |
| STSD= | 80.0 | Y= | -0.3959E-04 |
| STSD= | 90.0 | Y= | -0.1739E-04 |
| STSD= | 100.0 | Y= | 0.2000E-05 |
| AT ALFA = | -200.00 | | |
| STSD= | 50.0 | Y= | 0.4000E-04 |
| STSD= | 60.0 | Y= | 0.5507E-04 |
| STSD= | 70.0 | Y= | 0.6637E-04 |
| STSD= | 80.0 | Y= | 0.7767E-04 |
| STSD= | 90.0 | Y= | 0.8671E-04 |
| STSD= | 100.0 | Y= | 0.9500E-04 |
| AT ALFA = | -150.00 | | |
| STSD= | 50.0 | Y= | 0.2700E-03 |
| STSD= | 60.0 | Y= | 0.2508E-03 |
| STSD= | 70.0 | Y= | 0.2364E-03 |
| STSD= | 80.0 | Y= | 0.2221E-03 |
| STSD= | 90.0 | Y= | 0.2105E-03 |
| STSD= | 100.0 | Y= | 0.2000E-03 |
| AT ALFA = | -100.00 | | |
| STSD= | 50.0 | Y= | 0.5000E-03 |
| STSD= | 60.0 | Y= | 0.4507E-03 |
| STSD= | 70.0 | Y= | 0.4137E-03 |
| STSD= | 80.0 | Y= | 0.3767E-03 |
| STSD= | 90.0 | Y= | 0.3471E-03 |
| STSD= | 100.0 | Y= | 0.3200E-03 |
| AT ALFA = | -50.00 | | |
| STSD= | 50.0 | Y= | 0.7000E-03 |
| STSD= | 60.0 | Y= | 0.6225E-03 |
| STSD= | 70.0 | Y= | 0.5643E-03 |
| STSD= | 80.0 | Y= | 0.5062E-03 |
| STSD= | 90.0 | Y= | 0.4596E-03 |
| STSD= | 100.0 | Y= | 0.4170E-03 |

| | | | |
|-----------|--------|----|------------|
| AT ALFA = | 0.00 | | |
| STSD= | 50.0 | Y= | 0.3500E-03 |
| STSD= | 60.0 | Y= | 0.7500E-03 |
| STSD= | 70.0 | Y= | 0.6750E-03 |
| STSD= | 80.0 | Y= | 0.6000E-03 |
| STSD= | 90.0 | Y= | 0.5400E-03 |
| STSD= | 100.0 | Y= | 0.4850E-03 |
| AT ALFA = | 50.00 | | |
| STSD= | 50.0 | Y= | 0.9000E-03 |
| STSD= | 60.0 | Y= | 0.7344E-03 |
| STSD= | 70.0 | Y= | 0.6977E-03 |
| STSD= | 80.0 | Y= | 0.6110E-03 |
| STSD= | 90.0 | Y= | 0.5416E-03 |
| STSD= | 100.0 | Y= | 0.4780E-03 |
| AT ALFA = | 100.00 | | |
| STSD= | 50.0 | Y= | 0.8200E-03 |
| STSD= | 60.0 | Y= | 0.7159E-03 |
| STSD= | 70.0 | Y= | 0.6378E-03 |
| STSD= | 80.0 | Y= | 0.5597E-03 |
| STSD= | 90.0 | Y= | 0.4973E-03 |
| STSD= | 100.0 | Y= | 0.4400E-03 |
| AT ALFA = | 150.00 | | |
| STSD= | 50.0 | Y= | 0.6250E-03 |
| STSD= | 60.0 | Y= | 0.5579E-03 |
| STSD= | 70.0 | Y= | 0.5075E-03 |
| STSD= | 80.0 | Y= | 0.4572E-03 |
| STSD= | 90.0 | Y= | 0.4169E-03 |
| STSD= | 100.0 | Y= | 0.3800E-03 |
| AT ALFA = | 200.00 | | |
| STSD= | 50.0 | Y= | 0.4500E-03 |
| STSD= | 60.0 | Y= | 0.4089E-03 |
| STSD= | 70.0 | Y= | 0.3781E-03 |
| STSD= | 80.0 | Y= | 0.3473E-03 |
| STSD= | 90.0 | Y= | 0.3226E-03 |
| STSD= | 100.0 | Y= | 0.3000E-03 |
| AT ALFA = | 250.00 | | |
| STSD= | 50.0 | Y= | 0.3000E-03 |
| STSD= | 60.0 | Y= | 0.2781E-03 |
| STSD= | 70.0 | Y= | 0.2616E-03 |
| STSD= | 80.0 | Y= | 0.2452E-03 |
| STSD= | 90.0 | Y= | 0.2321E-03 |
| STSD= | 100.0 | Y= | 0.2200E-03 |

| | | | |
|-----------|--------|----|-------------|
| AT ALFA = | 300.00 | | |
| STSD= | 50.0 | Y= | 0.1700E-03 |
| STSD= | 60.0 | Y= | 0.1673E-03 |
| STSD= | 70.0 | Y= | 0.1652E-03 |
| STSD= | 80.0 | Y= | 0.1632E-03 |
| STSD= | 90.0 | Y= | 0.1615E-03 |
| STSD= | 100.0 | Y= | 0.1600E-03 |
| AT ALFA = | 350.00 | | |
| STSD= | 50.0 | Y= | 0.6000E-04 |
| STSD= | 60.0 | Y= | 0.6822E-04 |
| STSD= | 70.0 | Y= | 0.7438E-04 |
| STSD= | 80.0 | Y= | 0.8055E-04 |
| STSD= | 90.0 | Y= | 0.8548E-04 |
| STSD= | 100.0 | Y= | 0.9000E-04 |
| AT ALFA = | 400.00 | | |
| STSD= | 50.0 | Y= | -0.2000E-04 |
| STSD= | 60.0 | Y= | -0.3562E-05 |
| STSD= | 70.0 | Y= | 0.8767E-05 |
| STSD= | 80.0 | Y= | 0.2110E-04 |
| STSD= | 90.0 | Y= | 0.3096E-04 |
| STSD= | 100.0 | Y= | 0.4000E-04 |
| AT ALFA = | 450.00 | | |
| STSD= | 50.0 | Y= | -0.5000E-04 |
| STSD= | 60.0 | Y= | -0.3493E-04 |
| STSD= | 70.0 | Y= | -0.2363E-04 |
| STSD= | 80.0 | Y= | -0.1233E-04 |
| STSD= | 90.0 | Y= | -0.3238E-05 |
| STSD= | 100.0 | Y= | 0.5000E-05 |
| AT ALFA = | 500.00 | | |
| STSD= | 50.0 | Y= | -0.6000E-04 |
| STSD= | 60.0 | Y= | -0.4904E-04 |
| STSD= | 70.0 | Y= | -0.4082E-04 |
| STSD= | 80.0 | Y= | -0.3260E-04 |
| STSD= | 90.0 | Y= | -0.2603E-04 |
| STSD= | 100.0 | Y= | -0.2000E-04 |
| AT ALFA = | 550.00 | | |
| STSD= | 50.0 | Y= | -0.5000E-04 |
| STSD= | 60.0 | Y= | -0.4699E-04 |
| STSD= | 70.0 | Y= | -0.4473E-04 |
| STSD= | 80.0 | Y= | -0.4247E-04 |
| STSD= | 90.0 | Y= | -0.4066E-04 |
| STSD= | 100.0 | Y= | -0.3900E-04 |

| | | | |
|-----------|--------|----|-------------|
| AT ALFA = | 600.00 | | |
| STSD= | 50.0 | Y= | -0.1000E-04 |
| STSD= | 60.0 | Y= | -0.2096E-04 |
| STSD= | 70.0 | Y= | -0.2918E-04 |
| STSD= | 80.0 | Y= | -0.3740E-04 |
| STSD= | 90.0 | Y= | -0.4397E-04 |
| STSD= | 100.0 | Y= | -0.5000E-04 |

| | | | | | |
|-------|------|-------|--------|--------|-------------|
| STSD= | 50.0 | ALFA= | -550.0 | DY/DA= | -0.1300E-05 |
| STSD= | 50.0 | ALFA= | -500.0 | DY/DA= | -0.1400E-05 |
| STSD= | 50.0 | ALFA= | -450.0 | DY/DA= | -0.3000E-05 |
| STSD= | 50.0 | ALFA= | -400.0 | DY/DA= | 0.3000E-05 |
| STSD= | 50.0 | ALFA= | -350.0 | DY/DA= | 0.1400E-05 |
| STSD= | 50.0 | ALFA= | -300.0 | DY/DA= | 0.2200E-05 |
| STSD= | 50.0 | ALFA= | -250.0 | DY/DA= | 0.3000E-05 |
| STSD= | 50.0 | ALFA= | -200.0 | DY/DA= | 0.4000E-05 |
| STSD= | 50.0 | ALFA= | -150.0 | DY/DA= | 0.4600E-05 |
| STSD= | 50.0 | ALFA= | -100.0 | DY/DA= | 0.4300E-05 |
| STSD= | 50.0 | ALFA= | -50.0 | DY/DA= | 0.3500E-05 |
| STSD= | 50.0 | ALFA= | 0.0 | DY/DA= | 0.2000E-05 |
| STSD= | 50.0 | ALFA= | 50.0 | DY/DA= | -0.3000E-06 |
| STSD= | 50.0 | ALFA= | 100.0 | DY/DA= | -0.2750E-05 |
| STSD= | 50.0 | ALFA= | 150.0 | DY/DA= | -0.3700E-05 |
| STSD= | 50.0 | ALFA= | 200.0 | DY/DA= | -0.3250E-05 |
| STSD= | 50.0 | ALFA= | 250.0 | DY/DA= | -0.2800E-05 |
| STSD= | 50.0 | ALFA= | 300.0 | DY/DA= | -0.2400E-05 |
| STSD= | 50.0 | ALFA= | 350.0 | DY/DA= | -0.1900E-05 |
| STSD= | 50.0 | ALFA= | 400.0 | DY/DA= | -0.1100E-05 |
| STSD= | 50.0 | ALFA= | 450.0 | DY/DA= | -0.4000E-06 |
| STSD= | 50.0 | ALFA= | 500.0 | DY/DA= | 0.0000E-00 |
| STSD= | 50.0 | ALFA= | 550.0 | DY/DA= | 0.5000E-06 |
| STSD= | 60.0 | ALFA= | -550.0 | DY/DA= | -0.1499E-05 |
| STSD= | 60.0 | ALFA= | -500.0 | DY/DA= | -0.1112E-05 |
| STSD= | 60.0 | ALFA= | -450.0 | DY/DA= | -0.5753E-06 |
| STSD= | 60.0 | ALFA= | -400.0 | DY/DA= | 0.3548E-06 |
| STSD= | 60.0 | ALFA= | -350.0 | DY/DA= | 0.1253E-05 |
| STSD= | 60.0 | ALFA= | -300.0 | DY/DA= | 0.1932E-05 |
| STSD= | 60.0 | ALFA= | -250.0 | DY/DA= | 0.2630E-05 |
| STSD= | 60.0 | ALFA= | -200.0 | DY/DA= | 0.3447E-05 |
| STSD= | 60.0 | ALFA= | -150.0 | DY/DA= | 0.3956E-05 |
| STSD= | 60.0 | ALFA= | -100.0 | DY/DA= | 0.3716E-05 |
| STSD= | 60.0 | ALFA= | -50.0 | DY/DA= | 0.2993E-05 |
| STSD= | 60.0 | ALFA= | 0.0 | DY/DA= | 0.1619E-05 |
| STSD= | 60.0 | ALFA= | 50.0 | DY/DA= | -0.3411E-06 |
| STSD= | 60.0 | ALFA= | 100.0 | DY/DA= | -0.2265E-05 |
| STSD= | 60.0 | ALFA= | 150.0 | DY/DA= | -0.3070E-05 |
| STSD= | 60.0 | ALFA= | 200.0 | DY/DA= | -0.2798E-05 |
| STSD= | 60.0 | ALFA= | 250.0 | DY/DA= | -0.2416E-05 |
| STSD= | 60.0 | ALFA= | 300.0 | DY/DA= | -0.2099E-05 |
| STSD= | 60.0 | ALFA= | 350.0 | DY/DA= | -0.1708E-05 |
| STSD= | 60.0 | ALFA= | 400.0 | DY/DA= | -0.1032E-05 |
| STSD= | 60.0 | ALFA= | 450.0 | DY/DA= | -0.4548E-06 |
| STSD= | 60.0 | ALFA= | 500.0 | DY/DA= | -0.1205E-06 |
| STSD= | 60.0 | ALFA= | 550.0 | DY/DA= | 0.2303E-06 |

| | | | | | |
|-------|------|-------|--------|--------|-------------|
| STSD= | 70.0 | ALFA= | -550.0 | DY/DA= | -0.1273E-05 |
| STSD= | 70.0 | ALFA= | -500.0 | DY/DA= | -0.3966E-06 |
| STSD= | 70.0 | ALFA= | -450.0 | DY/DA= | -0.4068E-06 |
| STSD= | 70.0 | ALFA= | -400.0 | DY/DA= | 0.3959E-06 |
| STSD= | 70.0 | ALFA= | -350.0 | DY/DA= | 0.1151E-05 |
| STSD= | 70.0 | ALFA= | -300.0 | DY/DA= | 0.1730E-05 |
| STSD= | 70.0 | ALFA= | -250.0 | DY/DA= | 0.2353E-05 |
| STSD= | 70.0 | ALFA= | -200.0 | DY/DA= | 0.3032E-05 |
| STSD= | 70.0 | ALFA= | -150.0 | DY/DA= | 0.3473E-05 |
| STSD= | 70.0 | ALFA= | -100.0 | DY/DA= | 0.3279E-05 |
| STSD= | 70.0 | ALFA= | -50.0 | DY/DA= | 0.2613E-05 |
| STSD= | 70.0 | ALFA= | 0.0 | DY/DA= | 0.1334E-05 |
| STSD= | 70.0 | ALFA= | 50.0 | DY/DA= | -0.3719E-06 |
| STSD= | 70.0 | ALFA= | 100.0 | DY/DA= | -0.1901E-05 |
| STSD= | 70.0 | ALFA= | 150.0 | DY/DA= | -0.2597E-05 |
| STSD= | 70.0 | ALFA= | 200.0 | DY/DA= | -0.2459E-05 |
| STSD= | 70.0 | ALFA= | 250.0 | DY/DA= | -0.2129E-05 |
| STSD= | 70.0 | ALFA= | 300.0 | DY/DA= | -0.1873E-05 |
| STSD= | 70.0 | ALFA= | 350.0 | DY/DA= | -0.1564E-05 |
| STSD= | 70.0 | ALFA= | 400.0 | DY/DA= | -0.9801E-06 |
| STSD= | 70.0 | ALFA= | 450.0 | DY/DA= | -0.4959E-06 |
| STSD= | 70.0 | ALFA= | 500.0 | DY/DA= | -0.2110E-06 |
| STSD= | 70.0 | ALFA= | 550.0 | DY/DA= | 0.1164E-06 |
| STSD= | 80.0 | ALFA= | -550.0 | DY/DA= | -0.1047E-05 |
| STSD= | 80.0 | ALFA= | -500.0 | DY/DA= | -0.6608E-06 |
| STSD= | 80.0 | ALFA= | -450.0 | DY/DA= | -0.2384E-06 |
| STSD= | 80.0 | ALFA= | -400.0 | DY/DA= | 0.4370E-06 |
| STSD= | 80.0 | ALFA= | -350.0 | DY/DA= | 0.1044E-05 |
| STSD= | 80.0 | ALFA= | -300.0 | DY/DA= | 0.1529E-05 |
| STSD= | 80.0 | ALFA= | -250.0 | DY/DA= | 0.2075E-05 |
| STSD= | 80.0 | ALFA= | -200.0 | DY/DA= | 0.2316E-05 |
| STSD= | 80.0 | ALFA= | -150.0 | DY/DA= | 0.2390E-05 |
| STSD= | 80.0 | ALFA= | -100.0 | DY/DA= | 0.2341E-05 |
| STSD= | 80.0 | ALFA= | -50.0 | DY/DA= | 0.2233E-05 |
| STSD= | 80.0 | ALFA= | 0.0 | DY/DA= | 0.1048E-05 |
| STSD= | 80.0 | ALFA= | 50.0 | DY/DA= | -0.4027E-06 |
| STSD= | 80.0 | ALFA= | 100.0 | DY/DA= | -0.1538E-05 |
| STSD= | 80.0 | ALFA= | 150.0 | DY/DA= | -0.2125E-05 |
| STSD= | 80.0 | ALFA= | 200.0 | DY/DA= | -0.2120E-05 |
| STSD= | 80.0 | ALFA= | 250.0 | DY/DA= | -0.1841E-05 |
| STSD= | 80.0 | ALFA= | 300.0 | DY/DA= | -0.1647E-05 |
| STSD= | 80.0 | ALFA= | 350.0 | DY/DA= | -0.1421E-05 |
| STSD= | 80.0 | ALFA= | 400.0 | DY/DA= | -0.9233E-06 |
| STSD= | 80.0 | ALFA= | 450.0 | DY/DA= | -0.5370E-06 |
| STSD= | 80.0 | ALFA= | 500.0 | DY/DA= | -0.3014E-06 |
| STSD= | 80.0 | ALFA= | 550.0 | DY/DA= | -0.4795E-07 |

| | | | | | |
|-------|-------|-------|--------|--------|-------------|
| STSD= | 90.0 | ALFA= | -550.0 | DY/DA= | -0.8653E-06 |
| STSD= | 90.0 | ALFA= | -500.0 | DY/DA= | -0.5082E-06 |
| STSD= | 90.0 | ALFA= | -450.0 | DY/DA= | -0.1036E-06 |
| STSD= | 90.0 | ALFA= | -400.0 | DY/DA= | 0.4692E-06 |
| STSD= | 90.0 | ALFA= | -350.0 | DY/DA= | 0.9584E-06 |
| STSD= | 90.0 | ALFA= | -300.0 | DY/DA= | 0.1368E-05 |
| STSD= | 90.0 | ALFA= | -250.0 | DY/DA= | 0.1353E-05 |
| STSD= | 90.0 | ALFA= | -200.0 | DY/DA= | 0.2284E-05 |
| STSD= | 90.0 | ALFA= | -150.0 | DY/DA= | 0.2604E-05 |
| STSD= | 90.0 | ALFA= | -100.0 | DY/DA= | 0.2491E-05 |
| STSD= | 90.0 | ALFA= | -50.0 | DY/DA= | 0.1029E-05 |
| STSD= | 90.0 | ALFA= | 0.0 | DY/DA= | 0.8195E-06 |
| STSD= | 90.0 | ALFA= | 50.0 | DY/DA= | -0.4274E-06 |
| STSD= | 90.0 | ALFA= | 100.0 | DY/DA= | -0.1247E-05 |
| STSD= | 90.0 | ALFA= | 150.0 | DY/DA= | -0.1747E-05 |
| STSD= | 90.0 | ALFA= | 200.0 | DY/DA= | -0.1349E-05 |
| STSD= | 90.0 | ALFA= | 250.0 | DY/DA= | -0.1611E-05 |
| STSD= | 90.0 | ALFA= | 300.0 | DY/DA= | -0.1466E-05 |
| STSD= | 90.0 | ALFA= | 350.0 | DY/DA= | -0.1305E-05 |
| STSD= | 90.0 | ALFA= | 400.0 | DY/DA= | -0.8377E-06 |
| STSD= | 90.0 | ALFA= | 450.0 | DY/DA= | -0.5699E-06 |
| STSD= | 90.0 | ALFA= | 500.0 | DY/DA= | -0.3737E-06 |
| STSD= | 90.0 | ALFA= | 550.0 | DY/DA= | -0.1795E-06 |
| STSD= | 100.0 | ALFA= | -550.0 | DY/DA= | -0.7000E-06 |
| STSD= | 100.0 | ALFA= | -500.0 | DY/DA= | -0.3500E-06 |
| STSD= | 100.0 | ALFA= | -450.0 | DY/DA= | 0.2000E-07 |
| STSD= | 100.0 | ALFA= | -400.0 | DY/DA= | 0.5000E-06 |
| STSD= | 100.0 | ALFA= | -350.0 | DY/DA= | 0.8300E-06 |
| STSD= | 100.0 | ALFA= | -300.0 | DY/DA= | 0.1220E-05 |
| STSD= | 100.0 | ALFA= | -250.0 | DY/DA= | 0.1650E-05 |
| STSD= | 100.0 | ALFA= | -200.0 | DY/DA= | 0.1980E-05 |
| STSD= | 100.0 | ALFA= | -150.0 | DY/DA= | 0.2250E-05 |
| STSD= | 100.0 | ALFA= | -100.0 | DY/DA= | 0.2170E-05 |
| STSD= | 100.0 | ALFA= | -50.0 | DY/DA= | 0.1650E-05 |
| STSD= | 100.0 | ALFA= | 0.0 | DY/DA= | 0.6100E-06 |
| STSD= | 100.0 | ALFA= | 50.0 | DY/DA= | -0.4500E-06 |
| STSD= | 100.0 | ALFA= | 100.0 | DY/DA= | -0.9300E-06 |
| STSD= | 100.0 | ALFA= | 150.0 | DY/DA= | -0.1400E-05 |
| STSD= | 100.0 | ALFA= | 200.0 | DY/DA= | -0.1600E-05 |
| STSD= | 100.0 | ALFA= | 250.0 | DY/DA= | -0.1400E-05 |
| STSD= | 100.0 | ALFA= | 300.0 | DY/DA= | -0.1300E-05 |
| STSD= | 100.0 | ALFA= | 350.0 | DY/DA= | -0.1200E-05 |
| STSD= | 100.0 | ALFA= | 400.0 | DY/DA= | -0.8500E-06 |
| STSD= | 100.0 | ALFA= | 450.0 | DY/DA= | -0.6000E-06 |
| STSD= | 100.0 | ALFA= | 500.0 | DY/DA= | -0.4400E-06 |
| STSD= | 100.0 | ALFA= | 550.0 | DY/DA= | -0.3000E-06 |

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|-------|--------|-------|------|--------|------------|
| ALFA= | -600.0 | STSD= | 60.0 | DY/DB= | 0.1199E-05 |
| ALFA= | -600.0 | STSD= | 70.0 | DY/DB= | 0.1027E-05 |
| ALFA= | -600.0 | STSD= | 80.0 | DY/DB= | 0.9247E-06 |
| ALFA= | -600.0 | STSD= | 90.0 | DY/DB= | 0.7377E-06 |
| ALFA= | -550.0 | STSD= | 60.0 | DY/DB= | 0.2517E-05 |
| ALFA= | -550.0 | STSD= | 70.0 | DY/DB= | 0.2138E-05 |
| ALFA= | -550.0 | STSD= | 80.0 | DY/DB= | 0.1942E-05 |
| ALFA= | -550.0 | STSD= | 90.0 | DY/DB= | 0.1654E-05 |
| ALFA= | -500.0 | STSD= | 60.0 | DY/DB= | 0.3836E-05 |
| ALFA= | -500.0 | STSD= | 70.0 | DY/DB= | 0.3238E-05 |
| ALFA= | -500.0 | STSD= | 80.0 | DY/DB= | 0.2959E-05 |
| ALFA= | -500.0 | STSD= | 90.0 | DY/DB= | 0.2521E-05 |
| ALFA= | -450.0 | STSD= | 60.0 | DY/DB= | 0.5034E-05 |
| ALFA= | -450.0 | STSD= | 70.0 | DY/DB= | 0.4315E-05 |
| ALFA= | -450.0 | STSD= | 80.0 | DY/DB= | 0.3834E-05 |
| ALFA= | -450.0 | STSD= | 90.0 | DY/DB= | 0.3308E-05 |
| ALFA= | -400.0 | STSD= | 60.0 | DY/DB= | 0.5301E-05 |
| ALFA= | -400.0 | STSD= | 70.0 | DY/DB= | 0.4973E-05 |
| ALFA= | -400.0 | STSD= | 80.0 | DY/DB= | 0.4475E-05 |
| ALFA= | -400.0 | STSD= | 90.0 | DY/DB= | 0.3812E-05 |
| ALFA= | -350.0 | STSD= | 60.0 | DY/DB= | 0.5514E-05 |
| ALFA= | -350.0 | STSD= | 70.0 | DY/DB= | 0.4726E-05 |
| ALFA= | -350.0 | STSD= | 80.0 | DY/DB= | 0.4253E-05 |
| ALFA= | -350.0 | STSD= | 90.0 | DY/DB= | 0.3623E-05 |
| ALFA= | -300.0 | STSD= | 60.0 | DY/DB= | 0.4555E-05 |
| ALFA= | -300.0 | STSD= | 70.0 | DY/DB= | 0.3904E-05 |
| ALFA= | -300.0 | STSD= | 80.0 | DY/DB= | 0.3514E-05 |
| ALFA= | -300.0 | STSD= | 90.0 | DY/DB= | 0.2993E-05 |
| ALFA= | -250.0 | STSD= | 60.0 | DY/DB= | 0.3164E-05 |
| ALFA= | -250.0 | STSD= | 70.0 | DY/DB= | 0.2712E-05 |
| ALFA= | -250.0 | STSD= | 80.0 | DY/DB= | 0.2441E-05 |
| ALFA= | -250.0 | STSD= | 90.0 | DY/DB= | 0.2079E-05 |
| ALFA= | -200.0 | STSD= | 60.0 | DY/DB= | 0.1318E-05 |
| ALFA= | -200.0 | STSD= | 70.0 | DY/DB= | 0.1130E-05 |
| ALFA= | -200.0 | STSD= | 80.0 | DY/DB= | 0.1017E-05 |
| ALFA= | -200.0 | STSD= | 90.0 | DY/DB= | 0.8654E-06 |

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|-------|--------|-------|------|--------|-------------|
| ALFA= | -150.0 | STSD= | 60.0 | DY/DB= | -0.1678E-05 |
| ALFA= | -150.0 | STSD= | 70.0 | DY/DB= | -0.1438E-05 |
| ALFA= | -150.0 | STSD= | 80.0 | DY/DB= | -0.1295E-05 |
| ALFA= | -150.0 | STSD= | 90.0 | DY/DB= | -0.1103E-05 |
| ALFA= | -100.0 | STSD= | 60.0 | DY/DB= | -0.4315E-05 |
| ALFA= | -100.0 | STSD= | 70.0 | DY/DB= | -0.3609E-05 |
| ALFA= | -100.0 | STSD= | 80.0 | DY/DB= | -0.3329E-05 |
| ALFA= | -100.0 | STSD= | 90.0 | DY/DB= | -0.2836E-05 |
| ALFA= | -50.0 | STSD= | 60.0 | DY/DB= | -0.6734E-05 |
| ALFA= | -50.0 | STSD= | 70.0 | DY/DB= | -0.5815E-05 |
| ALFA= | -50.0 | STSD= | 80.0 | DY/DB= | -0.5234E-05 |
| ALFA= | -50.0 | STSD= | 90.0 | DY/DB= | -0.4458E-05 |
| ALFA= | 0.0 | STSD= | 60.0 | DY/DB= | -0.8750E-05 |
| ALFA= | 0.0 | STSD= | 70.0 | DY/DB= | -0.7500E-05 |
| ALFA= | 0.0 | STSD= | 80.0 | DY/DB= | -0.6750E-05 |
| ALFA= | 0.0 | STSD= | 90.0 | DY/DB= | -0.5750E-05 |
| ALFA= | 50.0 | STSD= | 60.0 | DY/DB= | -0.1012E-04 |
| ALFA= | 50.0 | STSD= | 70.0 | DY/DB= | -0.8671E-05 |
| ALFA= | 50.0 | STSD= | 80.0 | DY/DB= | -0.7804E-05 |
| ALFA= | 50.0 | STSD= | 90.0 | DY/DB= | -0.6643E-05 |
| ALFA= | 100.0 | STSD= | 60.0 | DY/DB= | -0.9110E-05 |
| ALFA= | 100.0 | STSD= | 70.0 | DY/DB= | -0.7808E-05 |
| ALFA= | 100.0 | STSD= | 80.0 | DY/DB= | -0.7027E-05 |
| ALFA= | 100.0 | STSD= | 90.0 | DY/DB= | -0.5936E-05 |
| ALFA= | 150.0 | STSD= | 60.0 | DY/DB= | -0.5873E-05 |
| ALFA= | 150.0 | STSD= | 70.0 | DY/DB= | -0.5034E-05 |
| ALFA= | 150.0 | STSD= | 80.0 | DY/DB= | -0.4531E-05 |
| ALFA= | 150.0 | STSD= | 90.0 | DY/DB= | -0.3860E-05 |
| ALFA= | 200.0 | STSD= | 60.0 | DY/DB= | -0.3596E-05 |
| ALFA= | 200.0 | STSD= | 70.0 | DY/DB= | -0.3082E-05 |
| ALFA= | 200.0 | STSD= | 80.0 | DY/DB= | -0.2774E-05 |
| ALFA= | 200.0 | STSD= | 90.0 | DY/DB= | -0.2363E-05 |
| ALFA= | 250.0 | STSD= | 60.0 | DY/DB= | -0.1918E-05 |
| ALFA= | 250.0 | STSD= | 70.0 | DY/DB= | -0.1644E-05 |
| ALFA= | 250.0 | STSD= | 80.0 | DY/DB= | -0.1479E-05 |
| ALFA= | 250.0 | STSD= | 90.0 | DY/DB= | -0.1260E-05 |
| ALFA= | 300.0 | STSD= | 60.0 | DY/DB= | -0.2307E-06 |
| ALFA= | 300.0 | STSD= | 70.0 | DY/DB= | -0.2055E-06 |
| ALFA= | 300.0 | STSD= | 80.0 | DY/DB= | -0.1849E-06 |
| ALFA= | 300.0 | STSD= | 90.0 | DY/DB= | -0.1575E-06 |

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|-------|-------|-------|------|--------|-------------|
| ALFA= | 350.0 | STSD= | 60.0 | DY/DB= | 0.7192E-06 |
| ALFA= | 350.0 | STSD= | 70.0 | DY/DB= | 0.6164E-06 |
| ALFA= | 350.0 | STSD= | 80.0 | DY/DB= | 0.5543E-06 |
| ALFA= | 350.0 | STSD= | 90.0 | DY/DB= | 0.4726E-06 |
| ALFA= | 400.0 | STSD= | 60.0 | DY/DB= | 0.1433E-05 |
| ALFA= | 400.0 | STSD= | 70.0 | DY/DB= | 0.1233E-05 |
| ALFA= | 400.0 | STSD= | 80.0 | DY/DB= | 0.1110E-05 |
| ALFA= | 400.0 | STSD= | 90.0 | DY/DB= | 0.9452E-06 |
| ALFA= | 450.0 | STSD= | 60.0 | DY/DB= | 0.1318E-05 |
| ALFA= | 450.0 | STSD= | 70.0 | DY/DB= | 0.1130E-05 |
| ALFA= | 450.0 | STSD= | 80.0 | DY/DB= | 0.1017E-05 |
| ALFA= | 450.0 | STSD= | 90.0 | DY/DB= | 0.8664E-06 |
| ALFA= | 500.0 | STSD= | 60.0 | DY/DB= | 0.9549E-06 |
| ALFA= | 500.0 | STSD= | 70.0 | DY/DB= | 0.8219E-06 |
| ALFA= | 500.0 | STSD= | 80.0 | DY/DB= | 0.7397E-06 |
| ALFA= | 500.0 | STSD= | 90.0 | DY/DB= | 0.6301E-06 |
| ALFA= | 550.0 | STSD= | 60.0 | DY/DB= | 0.2637E-06 |
| ALFA= | 550.0 | STSD= | 70.0 | DY/DB= | 0.2260E-06 |
| ALFA= | 550.0 | STSD= | 80.0 | DY/DB= | 0.2034E-06 |
| ALFA= | 550.0 | STSD= | 90.0 | DY/DB= | 0.1733E-06 |
| ALFA= | 600.0 | STSD= | 60.0 | DY/DB= | -0.9589E-06 |
| ALFA= | 600.0 | STSD= | 70.0 | DY/DB= | -0.8219E-06 |
| ALFA= | 600.0 | STSD= | 80.0 | DY/DB= | -0.7397E-06 |
| ALFA= | 600.0 | STSD= | 90.0 | DY/DB= | -0.6301E-06 |

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|-------------------|-------|----|------------|
| AT ALFA = -600.00 | | | |
| STSD= | 50.0 | Y= | 0.6000E-04 |
| STSD= | 60.0 | Y= | 0.6000E-04 |
| STSD= | 70.0 | Y= | 0.6000E-04 |
| STSD= | 80.0 | Y= | 0.6000E-04 |
| STSD= | 90.0 | Y= | 0.6000E-04 |
| STSD= | 100.0 | Y= | 0.6000E-04 |
| AT ALFA = -550.00 | | | |
| STSD= | 50.0 | Y= | 0.1100E-03 |
| STSD= | 60.0 | Y= | 0.1036E-03 |
| STSD= | 70.0 | Y= | 0.9786E-04 |
| STSD= | 80.0 | Y= | 0.9280E-04 |
| STSD= | 90.0 | Y= | 0.8890E-04 |
| STSD= | 100.0 | Y= | 0.8500E-04 |
| AT ALFA = -500.00 | | | |
| STSD= | 50.0 | Y= | 0.1390E-03 |
| STSD= | 60.0 | Y= | 0.1278E-03 |
| STSD= | 70.0 | Y= | 0.1176E-03 |
| STSD= | 80.0 | Y= | 0.1087E-03 |
| STSD= | 90.0 | Y= | 0.1019E-03 |
| STSD= | 100.0 | Y= | 0.9500E-04 |
| AT ALFA = -450.00 | | | |
| STSD= | 50.0 | Y= | 0.1530E-03 |
| STSD= | 60.0 | Y= | 0.1349E-03 |
| STSD= | 70.0 | Y= | 0.1185E-03 |
| STSD= | 80.0 | Y= | 0.1042E-03 |
| STSD= | 90.0 | Y= | 0.9308E-04 |
| STSD= | 100.0 | Y= | 0.8200E-04 |
| AT ALFA = -400.00 | | | |
| STSD= | 50.0 | Y= | 0.1400E-03 |
| STSD= | 60.0 | Y= | 0.1197E-03 |
| STSD= | 70.0 | Y= | 0.1012E-03 |
| STSD= | 80.0 | Y= | 0.2497E-04 |
| STSD= | 90.0 | Y= | 0.7249E-04 |
| STSD= | 100.0 | Y= | 0.6000E-04 |
| AT ALFA = -350.00 | | | |
| STSD= | 50.0 | Y= | 0.7000E-04 |
| STSD= | 60.0 | Y= | 0.5728E-04 |
| STSD= | 70.0 | Y= | 0.4572E-04 |
| STSD= | 80.0 | Y= | 0.3561E-04 |
| STSD= | 90.0 | Y= | 0.2780E-04 |
| STSD= | 100.0 | Y= | 0.2000E-04 |

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|-----------|---------|----|-------------|
| AT ALFA = | -300.00 | | |
| STSD= | 50.0 | Y= | -0.5000E-04 |
| STSD= | 60.0 | Y= | -0.4618E-04 |
| STSD= | 70.0 | Y= | -0.4272E-04 |
| STSD= | 80.0 | Y= | -0.3968E-04 |
| STSD= | 90.0 | Y= | -0.3734E-04 |
| STSD= | 100.0 | Y= | -0.3500E-04 |
| AT ALFA = | -250.00 | | |
| STSD= | 50.0 | Y= | -0.1750E-03 |
| STSD= | 60.0 | Y= | -0.1536E-03 |
| STSD= | 70.0 | Y= | -0.1342E-03 |
| STSD= | 80.0 | Y= | -0.1172E-03 |
| STSD= | 90.0 | Y= | -0.1041E-03 |
| STSD= | 100.0 | Y= | -0.9100E-04 |
| AT ALFA = | -200.00 | | |
| STSD= | 50.0 | Y= | -0.3000E-03 |
| STSD= | 60.0 | Y= | -0.2598E-03 |
| STSD= | 70.0 | Y= | -0.2233E-03 |
| STSD= | 80.0 | Y= | -0.1913E-03 |
| STSD= | 90.0 | Y= | -0.1667E-03 |
| STSD= | 100.0 | Y= | -0.1420E-03 |
| AT ALFA = | -150.00 | | |
| STSD= | 50.0 | Y= | -0.4100E-03 |
| STSD= | 60.0 | Y= | -0.3528E-03 |
| STSD= | 70.0 | Y= | -0.3008E-03 |
| STSD= | 80.0 | Y= | -0.2552E-03 |
| STSD= | 90.0 | Y= | -0.2201E-03 |
| STSD= | 100.0 | Y= | -0.1850E-03 |
| AT ALFA = | -100.00 | | |
| STSD= | 50.0 | Y= | -0.4420E-03 |
| STSD= | 60.0 | Y= | -0.3930E-03 |
| STSD= | 70.0 | Y= | -0.3294E-03 |
| STSD= | 80.0 | Y= | -0.2824E-03 |
| STSD= | 90.0 | Y= | -0.2462E-03 |
| STSD= | 100.0 | Y= | -0.2100E-03 |
| AT ALFA = | -50.00 | | |
| STSD= | 50.0 | Y= | -0.4350E-03 |
| STSD= | 60.0 | Y= | -0.3785E-03 |
| STSD= | 70.0 | Y= | -0.3272E-03 |
| STSD= | 80.0 | Y= | -0.2823E-03 |
| STSD= | 90.0 | Y= | -0.2476E-03 |
| STSD= | 100.0 | Y= | -0.2130E-03 |

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|-----------|--------|----|-------------|
| AT ALFA = | 0.00 | | |
| STSD= | 50.0 | Y= | -0.3740E-03 |
| STSD= | 60.0 | Y= | -0.3300E-03 |
| STSD= | 70.0 | Y= | -0.2900E-03 |
| STSD= | 80.0 | Y= | -0.2550E-03 |
| STSD= | 90.0 | Y= | -0.2280E-03 |
| STSD= | 100.0 | Y= | -0.2010E-03 |
| AT ALFA = | 50.00 | | |
| STSD= | 50.0 | Y= | -0.2800E-03 |
| STSD= | 60.0 | Y= | -0.2520E-03 |
| STSD= | 70.0 | Y= | -0.2266E-03 |
| STSD= | 80.0 | Y= | -0.2043E-03 |
| STSD= | 90.0 | Y= | -0.1872E-03 |
| STSD= | 100.0 | Y= | -0.1700E-03 |
| AT ALFA = | 100.00 | | |
| STSD= | 50.0 | Y= | -0.1500E-03 |
| STSD= | 60.0 | Y= | -0.1373E-03 |
| STSD= | 70.0 | Y= | -0.1257E-03 |
| STSD= | 80.0 | Y= | -0.1156E-03 |
| STSD= | 90.0 | Y= | -0.1078E-03 |
| STSD= | 100.0 | Y= | -0.1000E-03 |
| AT ALFA = | 150.00 | | |
| STSD= | 50.0 | Y= | -0.4500E-04 |
| STSD= | 60.0 | Y= | -0.4373E-04 |
| STSD= | 70.0 | Y= | -0.4257E-04 |
| STSD= | 80.0 | Y= | -0.4156E-04 |
| STSD= | 90.0 | Y= | -0.4078E-04 |
| STSD= | 100.0 | Y= | -0.4000E-04 |
| AT ALFA = | 200.00 | | |
| STSD= | 50.0 | Y= | 0.4200E-04 |
| STSD= | 60.0 | Y= | 0.3640E-04 |
| STSD= | 70.0 | Y= | 0.3132E-04 |
| STSD= | 80.0 | Y= | 0.2687E-04 |
| STSD= | 90.0 | Y= | 0.2343E-04 |
| STSD= | 100.0 | Y= | 0.2000E-04 |
| AT ALFA = | 250.00 | | |
| STSD= | 50.0 | Y= | 0.1100E-03 |
| STSD= | 60.0 | Y= | 0.9728E-04 |
| STSD= | 70.0 | Y= | 0.8572E-04 |
| STSD= | 80.0 | Y= | 0.7561E-04 |
| STSD= | 90.0 | Y= | 0.6780E-04 |
| STSD= | 100.0 | Y= | 0.6000E-04 |

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|-----------|--------|----|------------|
| AT ALFA = | 300.00 | | |
| STSD= | 50.0 | Y= | 0.1400E-03 |
| STSD= | 60.0 | Y= | 0.1247E-03 |
| STSD= | 70.0 | Y= | 0.1109E-03 |
| STSD= | 80.0 | Y= | 0.9873E-04 |
| STSD= | 90.0 | Y= | 0.8936E-04 |
| STSD= | 100.0 | Y= | 0.8000E-04 |
| AT ALFA = | 350.00 | | |
| STSD= | 50.0 | Y= | 0.1320E-03 |
| STSD= | 60.0 | Y= | 0.1193E-03 |
| STSD= | 70.0 | Y= | 0.1077E-03 |
| STSD= | 80.0 | Y= | 0.9761E-04 |
| STSD= | 90.0 | Y= | 0.8980E-04 |
| STSD= | 100.0 | Y= | 0.8200E-04 |
| AT ALFA = | 400.00 | | |
| STSD= | 50.0 | Y= | 0.1200E-03 |
| STSD= | 60.0 | Y= | 0.1060E-03 |
| STSD= | 70.0 | Y= | 0.9329E-04 |
| STSD= | 80.0 | Y= | 0.8217E-04 |
| STSD= | 90.0 | Y= | 0.7358E-04 |
| STSD= | 100.0 | Y= | 0.6500E-04 |
| AT ALFA = | 450.00 | | |
| STSD= | 50.0 | Y= | 0.1020E-03 |
| STSD= | 60.0 | Y= | 0.8750E-04 |
| STSD= | 70.0 | Y= | 0.7432E-04 |
| STSD= | 80.0 | Y= | 0.6279E-04 |
| STSD= | 90.0 | Y= | 0.5390E-04 |
| STSD= | 100.0 | Y= | 0.4500E-04 |
| AT ALFA = | 500.00 | | |
| STSD= | 50.0 | Y= | 0.7500E-04 |
| STSD= | 60.0 | Y= | 0.6355E-04 |
| STSD= | 70.0 | Y= | 0.5315E-04 |
| STSD= | 80.0 | Y= | 0.4405E-04 |
| STSD= | 90.0 | Y= | 0.3702E-04 |
| STSD= | 100.0 | Y= | 0.3000E-04 |
| AT ALFA = | 550.00 | | |
| STSD= | 50.0 | Y= | 0.4500E-04 |
| STSD= | 60.0 | Y= | 0.3864E-04 |
| STSD= | 70.0 | Y= | 0.3296E-04 |
| STSD= | 80.0 | Y= | 0.2760E-04 |
| STSD= | 90.0 | Y= | 0.2390E-04 |
| STSD= | 100.0 | Y= | 0.2000E-04 |

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|-----------|--------|----|------------|
| AT ALFA = | 600.00 | | |
| STSD= | 50.0 | Y= | 0.9000E-05 |
| STSD= | 60.0 | Y= | 0.1053E-04 |
| STSD= | 70.0 | Y= | 0.1191E-04 |
| STSD= | 80.0 | Y= | 0.1313E-04 |
| STSD= | 90.0 | Y= | 0.1406E-04 |
| STSD= | 100.0 | Y= | 0.1500E-04 |

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|-------|------|-------|--------|--------|-------------|
| STSD= | 50.0 | ALFA= | -550.0 | DY/DA= | 0.7200E-06 |
| STSD= | 50.0 | ALFA= | -500.0 | DY/DA= | 0.4300E-06 |
| STSD= | 50.0 | ALFA= | -450.0 | DY/DA= | 0.1000E-07 |
| STSD= | 50.0 | ALFA= | -400.0 | DY/DA= | -0.8300E-06 |
| STSD= | 50.0 | ALFA= | -350.0 | DY/DA= | -0.1900E-05 |
| STSD= | 50.0 | ALFA= | -300.0 | DY/DA= | -0.2450E-05 |
| STSD= | 50.0 | ALFA= | -250.0 | DY/DA= | -0.2500E-05 |
| STSD= | 50.0 | ALFA= | -200.0 | DY/DA= | -0.2350E-05 |
| STSD= | 50.0 | ALFA= | -150.0 | DY/DA= | -0.1420E-05 |
| STSD= | 50.0 | ALFA= | -100.0 | DY/DA= | -0.2500E-06 |
| STSD= | 50.0 | ALFA= | -50.0 | DY/DA= | 0.6300E-06 |
| STSD= | 50.0 | ALFA= | 0.0 | DY/DA= | 0.1550E-05 |
| STSD= | 50.0 | ALFA= | 50.0 | DY/DA= | 0.2240E-05 |
| STSD= | 50.0 | ALFA= | 100.0 | DY/DA= | 0.2350E-05 |
| STSD= | 50.0 | ALFA= | 150.0 | DY/DA= | 0.1920E-05 |
| STSD= | 50.0 | ALFA= | 200.0 | DY/DA= | 0.1550E-05 |
| STSD= | 50.0 | ALFA= | 250.0 | DY/DA= | 0.9300E-06 |
| STSD= | 50.0 | ALFA= | 300.0 | DY/DA= | 0.2200E-06 |
| STSD= | 50.0 | ALFA= | 350.0 | DY/DA= | -0.2000E-06 |
| STSD= | 50.0 | ALFA= | 400.0 | DY/DA= | -0.3000E-06 |
| STSD= | 50.0 | ALFA= | 450.0 | DY/DA= | -0.4500E-06 |
| STSD= | 50.0 | ALFA= | 500.0 | DY/DA= | -0.5700E-06 |
| STSD= | 50.0 | ALFA= | 550.0 | DY/DA= | -0.6600E-06 |
| STSD= | 60.0 | ALFA= | -550.0 | DY/DA= | 0.6781E-06 |
| STSD= | 60.0 | ALFA= | -500.0 | DY/DA= | 0.3130E-06 |
| STSD= | 60.0 | ALFA= | -450.0 | DY/DA= | -0.8156E-07 |
| STSD= | 60.0 | ALFA= | -400.0 | DY/DA= | -0.7766E-06 |
| STSD= | 60.0 | ALFA= | -350.0 | DY/DA= | -0.1653E-05 |
| STSD= | 60.0 | ALFA= | -300.0 | DY/DA= | -0.2109E-05 |
| STSD= | 60.0 | ALFA= | -250.0 | DY/DA= | -0.2136E-05 |
| STSD= | 60.0 | ALFA= | -200.0 | DY/DA= | -0.1991E-05 |
| STSD= | 60.0 | ALFA= | -150.0 | DY/DA= | -0.1232E-05 |
| STSD= | 60.0 | ALFA= | -100.0 | DY/DA= | -0.2575E-06 |
| STSD= | 60.0 | ALFA= | -50.0 | DY/DA= | 0.5299E-06 |
| STSD= | 60.0 | ALFA= | 0.0 | DY/DA= | 0.1265E-05 |
| STSD= | 60.0 | ALFA= | 50.0 | DY/DA= | 0.1927E-05 |
| STSD= | 60.0 | ALFA= | 100.0 | DY/DA= | 0.2093E-05 |
| STSD= | 60.0 | ALFA= | 150.0 | DY/DA= | 0.1737E-05 |
| STSD= | 60.0 | ALFA= | 200.0 | DY/DA= | 0.1410E-05 |
| STSD= | 60.0 | ALFA= | 250.0 | DY/DA= | 0.3834E-06 |
| STSD= | 60.0 | ALFA= | 300.0 | DY/DA= | 0.2200E-06 |
| STSD= | 60.0 | ALFA= | 350.0 | DY/DA= | -0.1873E-06 |
| STSD= | 60.0 | ALFA= | 400.0 | DY/DA= | -0.3178E-06 |
| STSD= | 60.0 | ALFA= | 450.0 | DY/DA= | -0.4246E-06 |
| STSD= | 60.0 | ALFA= | 500.0 | DY/DA= | -0.4886E-06 |
| STSD= | 60.0 | ALFA= | 550.0 | DY/DA= | -0.5303E-06 |

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| STSD= | 70.0 | ALFA= | -550.0 | DY/DA= | 0.5764E-06 |
| STSD= | 70.0 | ALFA= | -500.0 | DY/DA= | 0.2066E-06 |
| STSD= | 70.0 | ALFA= | -450.0 | DY/DA= | -0.1643E-06 |
| STSD= | 70.0 | ALFA= | -400.0 | DY/DA= | -0.7280E-06 |
| STSD= | 70.0 | ALFA= | -350.0 | DY/DA= | -0.1439E-05 |
| STSD= | 70.0 | ALFA= | -300.0 | DY/DA= | -0.1799E-05 |
| STSD= | 70.0 | ALFA= | -250.0 | DY/DA= | -0.1906E-05 |
| STSD= | 70.0 | ALFA= | -200.0 | DY/DA= | -0.1666E-05 |
| STSD= | 70.0 | ALFA= | -150.0 | DY/DA= | -0.1061E-05 |
| STSD= | 70.0 | ALFA= | -100.0 | DY/DA= | -0.2646E-06 |
| STSD= | 70.0 | ALFA= | -50.0 | DY/DA= | 0.3935E-06 |
| STSD= | 70.0 | ALFA= | 0.0 | DY/DA= | 0.1006E-05 |
| STSD= | 70.0 | ALFA= | 50.0 | DY/DA= | 0.1643E-05 |
| STSD= | 70.0 | ALFA= | 100.0 | DY/DA= | 0.1340E-05 |
| STSD= | 70.0 | ALFA= | 150.0 | DY/DA= | 0.1570E-05 |
| STSD= | 70.0 | ALFA= | 200.0 | DY/DA= | 0.1283E-05 |
| STSD= | 70.0 | ALFA= | 250.0 | DY/DA= | 0.7955E-06 |
| STSD= | 70.0 | ALFA= | 300.0 | DY/DA= | 0.2200E-06 |
| STSD= | 70.0 | ALFA= | 350.0 | DY/DA= | -0.1757E-06 |
| STSD= | 70.0 | ALFA= | 400.0 | DY/DA= | -0.3340E-06 |
| STSD= | 70.0 | ALFA= | 450.0 | DY/DA= | -0.4014E-06 |
| STSD= | 70.0 | ALFA= | 500.0 | DY/DA= | -0.4146E-06 |
| STSD= | 70.0 | ALFA= | 550.0 | DY/DA= | -0.4124E-06 |
| STSD= | 80.0 | ALFA= | -550.0 | DY/DA= | 0.4373E-06 |
| STSD= | 80.0 | ALFA= | -500.0 | DY/DA= | 0.1136E-06 |
| STSD= | 80.0 | ALFA= | -450.0 | DY/DA= | -0.2376E-06 |
| STSD= | 80.0 | ALFA= | -400.0 | DY/DA= | -0.6355E-06 |
| STSD= | 80.0 | ALFA= | -350.0 | DY/DA= | -0.1247E-05 |
| STSD= | 80.0 | ALFA= | -300.0 | DY/DA= | -0.1528E-05 |
| STSD= | 80.0 | ALFA= | -250.0 | DY/DA= | -0.1516E-05 |
| STSD= | 80.0 | ALFA= | -200.0 | DY/DA= | -0.1380E-05 |
| STSD= | 80.0 | ALFA= | -150.0 | DY/DA= | -0.9110E-06 |
| STSD= | 80.0 | ALFA= | -100.0 | DY/DA= | -0.2706E-06 |
| STSD= | 80.0 | ALFA= | -50.0 | DY/DA= | 0.2742E-06 |
| STSD= | 80.0 | ALFA= | 0.0 | DY/DA= | 0.7796E-06 |
| STSD= | 80.0 | ALFA= | 50.0 | DY/DA= | 0.1394E-05 |
| STSD= | 80.0 | ALFA= | 100.0 | DY/DA= | 0.1628E-05 |
| STSD= | 80.0 | ALFA= | 150.0 | DY/DA= | 0.1425E-05 |
| STSD= | 80.0 | ALFA= | 200.0 | DY/DA= | 0.1172E-05 |
| STSD= | 80.0 | ALFA= | 250.0 | DY/DA= | 0.7186E-06 |
| STSD= | 80.0 | ALFA= | 300.0 | DY/DA= | 0.2200E-06 |
| STSD= | 80.0 | ALFA= | 350.0 | DY/DA= | -0.1656E-06 |
| STSD= | 80.0 | ALFA= | 400.0 | DY/DA= | -0.3482E-06 |
| STSD= | 80.0 | ALFA= | 450.0 | DY/DA= | -0.3312E-06 |
| STSD= | 80.0 | ALFA= | 500.0 | DY/DA= | -0.3490E-06 |
| STSD= | 80.0 | ALFA= | 550.0 | DY/DA= | -0.3092E-06 |

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|-------|-------|-------|--------|--------|-------------|
| STSD= | 90.0 | ALFA= | -550.0 | DY/DA= | 0.4187E-06 |
| STSD= | 90.0 | ALFA= | -500.0 | DY/DA= | 0.4179E-07 |
| STSD= | 90.0 | ALFA= | -450.0 | DY/DA= | -0.2958E-06 |
| STSD= | 90.0 | ALFA= | -400.0 | DY/DA= | -0.6528E-06 |
| STSD= | 90.0 | ALFA= | -350.0 | DY/DA= | -0.1098E-05 |
| STSD= | 90.0 | ALFA= | -300.0 | DY/DA= | -0.1319E-05 |
| STSD= | 90.0 | ALFA= | -250.0 | DY/DA= | -0.1293E-05 |
| STSD= | 90.0 | ALFA= | -200.0 | DY/DA= | -0.1160E-05 |
| STSD= | 90.0 | ALFA= | -150.0 | DY/DA= | -0.7955E-06 |
| STSD= | 90.0 | ALFA= | -100.0 | DY/DA= | -0.2753E-06 |
| STSD= | 90.0 | ALFA= | -50.0 | DY/DA= | 0.1821E-06 |
| STSD= | 90.0 | ALFA= | 0.0 | DY/DA= | 0.6048E-06 |
| STSD= | 90.0 | ALFA= | 50.0 | DY/DA= | 0.1202E-05 |
| STSD= | 90.0 | ALFA= | 100.0 | DY/DA= | 0.1464E-05 |
| STSD= | 90.0 | ALFA= | 150.0 | DY/DA= | 0.1312E-05 |
| STSD= | 90.0 | ALFA= | 200.0 | DY/DA= | 0.1085E-05 |
| STSD= | 90.0 | ALFA= | 250.0 | DY/DA= | 0.5593E-06 |
| STSD= | 90.0 | ALFA= | 300.0 | DY/DA= | 0.2200E-06 |
| STSD= | 90.0 | ALFA= | 350.0 | DY/DA= | -0.1578E-06 |
| STSD= | 90.0 | ALFA= | 400.0 | DY/DA= | -0.3591E-06 |
| STSD= | 90.0 | ALFA= | 450.0 | DY/DA= | -0.3656E-06 |
| STSD= | 90.0 | ALFA= | 500.0 | DY/DA= | -0.2999E-06 |
| STSD= | 90.0 | ALFA= | 550.0 | DY/DA= | -0.2296E-06 |
| STSD= | 100.0 | ALFA= | -550.0 | DY/DA= | 0.3500E-06 |
| STSD= | 100.0 | ALFA= | -500.0 | DY/DA= | -0.3000E-07 |
| STSD= | 100.0 | ALFA= | -450.0 | DY/DA= | -0.3500E-06 |
| STSD= | 100.0 | ALFA= | -400.0 | DY/DA= | -0.6200E-06 |
| STSD= | 100.0 | ALFA= | -350.0 | DY/DA= | -0.9500E-06 |
| STSD= | 100.0 | ALFA= | -300.0 | DY/DA= | -0.1110E-05 |
| STSD= | 100.0 | ALFA= | -250.0 | DY/DA= | -0.1070E-05 |
| STSD= | 100.0 | ALFA= | -200.0 | DY/DA= | -0.9400E-06 |
| STSD= | 100.0 | ALFA= | -150.0 | DY/DA= | -0.6800E-06 |
| STSD= | 100.0 | ALFA= | -100.0 | DY/DA= | -0.2300E-06 |
| STSD= | 100.0 | ALFA= | -50.0 | DY/DA= | 0.9000E-07 |
| STSD= | 100.0 | ALFA= | 0.0 | DY/DA= | 0.4300E-06 |
| STSD= | 100.0 | ALFA= | 50.0 | DY/DA= | 0.1010E-05 |
| STSD= | 100.0 | ALFA= | 100.0 | DY/DA= | 0.1300E-05 |
| STSD= | 100.0 | ALFA= | 150.0 | DY/DA= | 0.1200E-05 |
| STSD= | 100.0 | ALFA= | 200.0 | DY/DA= | 0.1000E-05 |
| STSD= | 100.0 | ALFA= | 250.0 | DY/DA= | 0.6000E-06 |
| STSD= | 100.0 | ALFA= | 300.0 | DY/DA= | 0.2200E-06 |
| STSD= | 100.0 | ALFA= | 350.0 | DY/DA= | -0.1500E-06 |
| STSD= | 100.0 | ALFA= | 400.0 | DY/DA= | -0.3700E-06 |
| STSD= | 100.0 | ALFA= | 450.0 | DY/DA= | -0.3500E-06 |
| STSD= | 100.0 | ALFA= | 500.0 | DY/DA= | -0.2500E-06 |
| STSD= | 100.0 | ALFA= | 550.0 | DY/DA= | -0.1500E-06 |

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|-------|--------|-------|------|--------|-------------|
| ALFA= | -600.0 | STSD= | 60.0 | DY/DB= | 0.0000E-00 |
| ALFA= | -600.0 | STSD= | 70.0 | DY/DB= | 0.0000E-00 |
| ALFA= | -600.0 | STSD= | 80.0 | DY/DB= | 0.0000E-00 |
| ALFA= | -600.0 | STSD= | 90.0 | DY/DB= | 0.0000E-00 |
| ALFA= | -550.0 | STSD= | 60.0 | DY/DB= | -0.6069E-06 |
| ALFA= | -550.0 | STSD= | 70.0 | DY/DB= | -0.5419E-06 |
| ALFA= | -550.0 | STSD= | 80.0 | DY/DB= | -0.4430E-06 |
| ALFA= | -550.0 | STSD= | 90.0 | DY/DB= | -0.3902E-06 |
| ALFA= | -500.0 | STSD= | 60.0 | DY/DB= | -0.1068E-05 |
| ALFA= | -500.0 | STSD= | 70.0 | DY/DB= | -0.9538E-06 |
| ALFA= | -500.0 | STSD= | 80.0 | DY/DB= | -0.7834E-06 |
| ALFA= | -500.0 | STSD= | 90.0 | DY/DB= | -0.6867E-06 |
| ALFA= | -450.0 | STSD= | 60.0 | DY/DB= | -0.1724E-05 |
| ALFA= | -450.0 | STSD= | 70.0 | DY/DB= | -0.1539E-05 |
| ALFA= | -450.0 | STSD= | 80.0 | DY/DB= | -0.1272E-05 |
| ALFA= | -450.0 | STSD= | 90.0 | DY/DB= | -0.1108E-05 |
| ALFA= | -400.0 | STSD= | 60.0 | DY/DB= | -0.1942E-05 |
| ALFA= | -400.0 | STSD= | 70.0 | DY/DB= | -0.1734E-05 |
| ALFA= | -400.0 | STSD= | 80.0 | DY/DB= | -0.1434E-05 |
| ALFA= | -400.0 | STSD= | 90.0 | DY/DB= | -0.1249E-05 |
| ALFA= | -350.0 | STSD= | 60.0 | DY/DB= | -0.1214E-05 |
| ALFA= | -350.0 | STSD= | 70.0 | DY/DB= | -0.1034E-05 |
| ALFA= | -350.0 | STSD= | 80.0 | DY/DB= | -0.8060E-06 |
| ALFA= | -350.0 | STSD= | 90.0 | DY/DB= | -0.7803E-06 |
| ALFA= | -300.0 | STSD= | 60.0 | DY/DB= | -0.3642E-06 |
| ALFA= | -300.0 | STSD= | 70.0 | DY/DB= | 0.3251E-06 |
| ALFA= | -300.0 | STSD= | 80.0 | DY/DB= | 0.2638E-06 |
| ALFA= | -300.0 | STSD= | 90.0 | DY/DB= | 0.2341E-06 |
| ALFA= | -250.0 | STSD= | 60.0 | DY/DB= | 0.2039E-05 |
| ALFA= | -250.0 | STSD= | 70.0 | DY/DB= | 0.1821E-05 |
| ALFA= | -250.0 | STSD= | 80.0 | DY/DB= | 0.1505E-05 |
| ALFA= | -250.0 | STSD= | 90.0 | DY/DB= | 0.1311E-05 |
| ALFA= | -200.0 | STSD= | 60.0 | DY/DB= | 0.3336E-05 |
| ALFA= | -200.0 | STSD= | 70.0 | DY/DB= | 0.3425E-05 |
| ALFA= | -200.0 | STSD= | 80.0 | DY/DB= | 0.2831E-05 |
| ALFA= | -200.0 | STSD= | 90.0 | DY/DB= | 0.2466E-05 |

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|-------|--------|-------|------|--------|-------------|
| ALFA= | -150.0 | STSD= | 60.0 | DY/DB= | 0.5432E-05 |
| ALFA= | -150.0 | STSD= | 70.0 | DY/DB= | 0.4377E-05 |
| ALFA= | -150.0 | STSD= | 80.0 | DY/DB= | 0.4032E-05 |
| ALFA= | -150.0 | STSD= | 90.0 | DY/DB= | 0.3512E-05 |
| ALFA= | -100.0 | STSD= | 60.0 | DY/DB= | 0.5632E-05 |
| ALFA= | -100.0 | STSD= | 70.0 | DY/DB= | 0.5029E-05 |
| ALFA= | -100.0 | STSD= | 80.0 | DY/DB= | 0.4157E-05 |
| ALFA= | -100.0 | STSD= | 90.0 | DY/DB= | 0.3621E-05 |
| ALFA= | -50.0 | STSD= | 60.0 | DY/DB= | 0.5390E-05 |
| ALFA= | -50.0 | STSD= | 70.0 | DY/DB= | 0.4812E-05 |
| ALFA= | -50.0 | STSD= | 80.0 | DY/DB= | 0.3978E-05 |
| ALFA= | -50.0 | STSD= | 90.0 | DY/DB= | 0.3465E-05 |
| ALFA= | 0.0 | STSD= | 60.0 | DY/DB= | 0.4200E-05 |
| ALFA= | 0.0 | STSD= | 70.0 | DY/DB= | 0.3750E-05 |
| ALFA= | 0.0 | STSD= | 80.0 | DY/DB= | 0.3100E-05 |
| ALFA= | 0.0 | STSD= | 90.0 | DY/DB= | 0.2700E-05 |
| ALFA= | 50.0 | STSD= | 60.0 | DY/DB= | 0.2671E-05 |
| ALFA= | 50.0 | STSD= | 70.0 | DY/DB= | 0.2384E-05 |
| ALFA= | 50.0 | STSD= | 80.0 | DY/DB= | 0.1971E-05 |
| ALFA= | 50.0 | STSD= | 90.0 | DY/DB= | 0.1717E-05 |
| ALFA= | 100.0 | STSD= | 60.0 | DY/DB= | 0.1214E-05 |
| ALFA= | 100.0 | STSD= | 70.0 | DY/DB= | 0.1034E-05 |
| ALFA= | 100.0 | STSD= | 80.0 | DY/DB= | 0.8960E-06 |
| ALFA= | 100.0 | STSD= | 90.0 | DY/DB= | 0.7303E-06 |
| ALFA= | 150.0 | STSD= | 60.0 | DY/DB= | 0.1214E-06 |
| ALFA= | 150.0 | STSD= | 70.0 | DY/DB= | 0.1034E-06 |
| ALFA= | 150.0 | STSD= | 80.0 | DY/DB= | 0.8960E-07 |
| ALFA= | 150.0 | STSD= | 90.0 | DY/DB= | 0.7303E-07 |
| ALFA= | 200.0 | STSD= | 60.0 | DY/DB= | -0.5341E-06 |
| ALFA= | 200.0 | STSD= | 70.0 | DY/DB= | -0.4769E-06 |
| ALFA= | 200.0 | STSD= | 80.0 | DY/DB= | -0.3942E-06 |
| ALFA= | 200.0 | STSD= | 90.0 | DY/DB= | -0.3434E-06 |
| ALFA= | 250.0 | STSD= | 60.0 | DY/DB= | -0.1214E-05 |
| ALFA= | 250.0 | STSD= | 70.0 | DY/DB= | -0.1034E-05 |
| ALFA= | 250.0 | STSD= | 80.0 | DY/DB= | -0.8960E-06 |
| ALFA= | 250.0 | STSD= | 90.0 | DY/DB= | -0.7303E-06 |
| ALFA= | 300.0 | STSD= | 60.0 | DY/DB= | -0.1457E-05 |
| ALFA= | 300.0 | STSD= | 70.0 | DY/DB= | -0.1301E-05 |
| ALFA= | 300.0 | STSD= | 80.0 | DY/DB= | -0.1075E-05 |
| ALFA= | 300.0 | STSD= | 90.0 | DY/DB= | -0.9364E-06 |

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|-------|-------|-------|------|--------|-------------|
| ALFA= | 350.0 | STSD= | 60.0 | DY/DB= | -0.1214E-05 |
| ALFA= | 350.0 | STSD= | 70.0 | DY/DB= | -0.1034E-05 |
| ALFA= | 350.0 | STSD= | 80.0 | DY/DB= | -0.8760E-06 |
| ALFA= | 350.0 | STSD= | 90.0 | DY/DB= | -0.7803E-06 |
| ALFA= | 400.0 | STSD= | 60.0 | DY/DB= | -0.1335E-05 |
| ALFA= | 400.0 | STSD= | 70.0 | DY/DB= | -0.1192E-05 |
| ALFA= | 400.0 | STSD= | 80.0 | DY/DB= | -0.9356E-06 |
| ALFA= | 400.0 | STSD= | 90.0 | DY/DB= | -0.8534E-06 |
| ALFA= | 450.0 | STSD= | 60.0 | DY/DB= | -0.1334E-05 |
| ALFA= | 450.0 | STSD= | 70.0 | DY/DB= | -0.1236E-05 |
| ALFA= | 450.0 | STSD= | 80.0 | DY/DB= | -0.1021E-05 |
| ALFA= | 450.0 | STSD= | 90.0 | DY/DB= | -0.8836E-06 |
| ALFA= | 500.0 | STSD= | 60.0 | DY/DB= | -0.1002E-05 |
| ALFA= | 500.0 | STSD= | 70.0 | DY/DB= | -0.9754E-06 |
| ALFA= | 500.0 | STSD= | 80.0 | DY/DB= | -0.8064E-06 |
| ALFA= | 500.0 | STSD= | 90.0 | DY/DB= | -0.7023E-06 |
| ALFA= | 550.0 | STSD= | 60.0 | DY/DB= | -0.6069E-06 |
| ALFA= | 550.0 | STSD= | 70.0 | DY/DB= | -0.5419E-06 |
| ALFA= | 550.0 | STSD= | 80.0 | DY/DB= | -0.4430E-06 |
| ALFA= | 550.0 | STSD= | 90.0 | DY/DB= | -0.3902E-06 |
| ALFA= | 600.0 | STSD= | 60.0 | DY/DB= | 0.1457E-06 |
| ALFA= | 600.0 | STSD= | 70.0 | DY/DB= | 0.1301E-06 |
| ALFA= | 600.0 | STSD= | 80.0 | DY/DB= | 0.1075E-06 |
| ALFA= | 600.0 | STSD= | 90.0 | DY/DB= | 0.9364E-07 |

Appendix III

PROGRAM DETERM

- I. Discussion
- II. Input/Output Format
- III. Flow Chart
- IV. Program Listing
- V. Sample Results

Appendix III

PROGRAM DETERM

I. Discussion

Program DETERM determines the hydrodynamic derivatives with respect to propeller speed, n , which were not available in the literature surveyed. It also determines the variation of interaction effects with velocity, u . Thus the outputs are X_n , Y_n and N_n for Ship A; Y_u and N_u for Ship A; and Y_u and N_u for Ship A in Ship B's position. These latter values, when used as inputs in program SOLVE are modified to their appropriate values for Ship B in Ship B's position. (X_n , Y_n and N_n are position-independent.)

The values of X_n , Y_n and N_n are calculated by using the known values of X , Y and N exerted by the propeller at equilibrium speed in open water and assuming that propeller forces and moments vary as the square of propeller speed. The derivatives with respect to u also are calculated by taking the known values of Y and N at equilibrium and assuming that they vary as the square of the velocity, u . (The procedure followed in calculating these quantities is described in detail in Chapter III, PROCEDURES.)

II. Input/Output Format

Program DETERM's input is ordered as follows:

| ITEM | PROGRAM SYMBOL | CARD | COLUMNS | FORMAT |
|--------------------------------------|-------------------|------|---------|--------|
| LBP Ship A | ALBP | 1 | 1-10 | F10.2 |
| Beam Ship A | ABEAM | 1 | 11-20 | F10.2 |
| Speed at 1 kt. below equil. speed | V(1) | 2 | 1-10 | F10.2 |
| RPM at this speed | RPM(1) | 2 | 11-20 | F10.2 |
| EHP at this speed | EHP(1) | 2 | 21-30 | F10.2 |
| Equil. speed (kts.) | V(2) | 3 | 1-10 | F10.2 |
| RPM at equil. | RPM(2) | 3 | 11-20 | F10.2 |
| EHP at equil. | EHP(2) | 3 | 21-30 | F10.2 |
| Speed at 1 kt. above equil. speed | V(3) | 4 | 1-10 | F10.2 |
| RPM at this speed | RPM(3) | 4 | 11-20 | F10.2 |
| EHP at this speed | EHP(3) | 4 | 21-30 | F10.2 |
| # positions examined | NPOSIT | 5 | 1-4 | I4 |
| Long. separation at each position | ALFAO(I) | 6 | 1-10 | F10.2 |
| Lat. separation at each position | STSD(I) | 6 | 11-20 | F10.2 |
| (one card for each position) | | | | |
| Y force at each position, Ship A | AY(M) | 7 | 1-14 | E14.4 |
| N moment at each position, Ship A | AN(M) | 7 | 15-28 | E14.4 |

(one card for each position)

| ITEM | PROGRAM SYMBOL | CARD | COLUMNS | FORMAT |
|--|-------------------|------|---------|--------|
| Y force on Ship A in Ship B position | BY(N) | 8 | 1-14 | E14.4 |
| N moment on Ship A in Ship B position | BN(N) | 8 | 15-28 | E14.4 |
| (one card for each position) | | | | |
| Y_{δ} Ship A | AYDEL | 9 | 1-14 | E14.4 |
| N_{δ} Ship A | ANDEL | 9 | 15-28 | E14.4 |
| δ at equil, Ship A | ADELO | 9 | 29-38 | F10.2 |

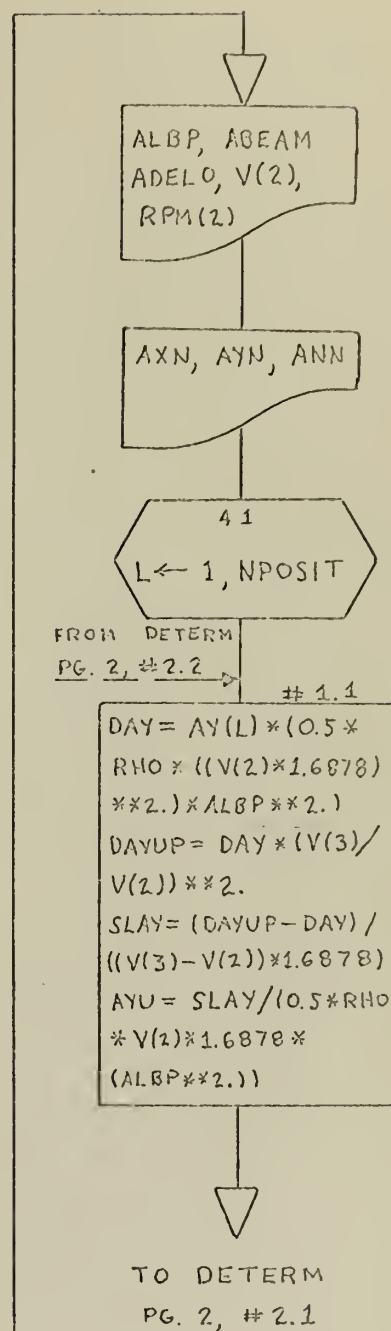
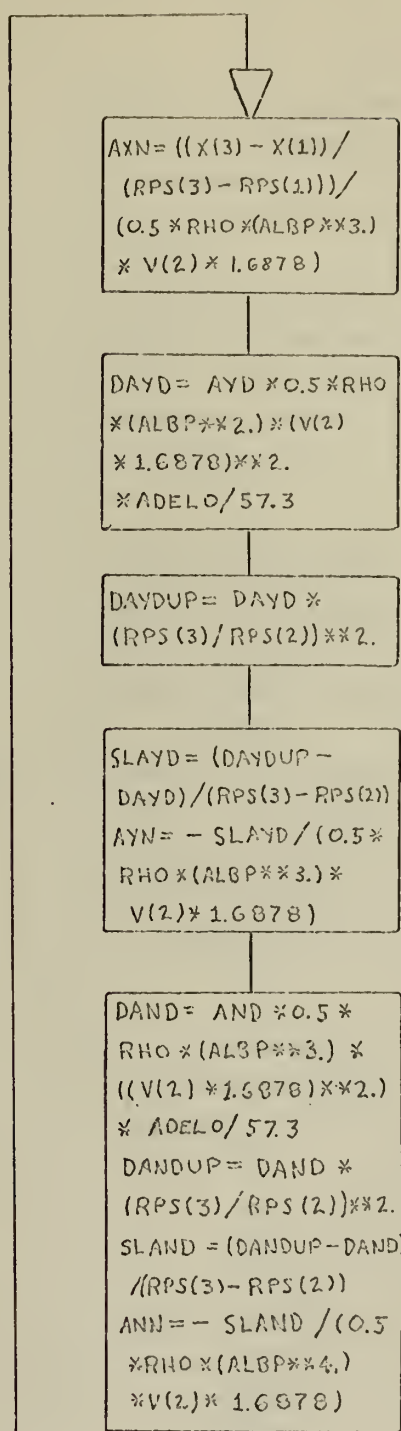
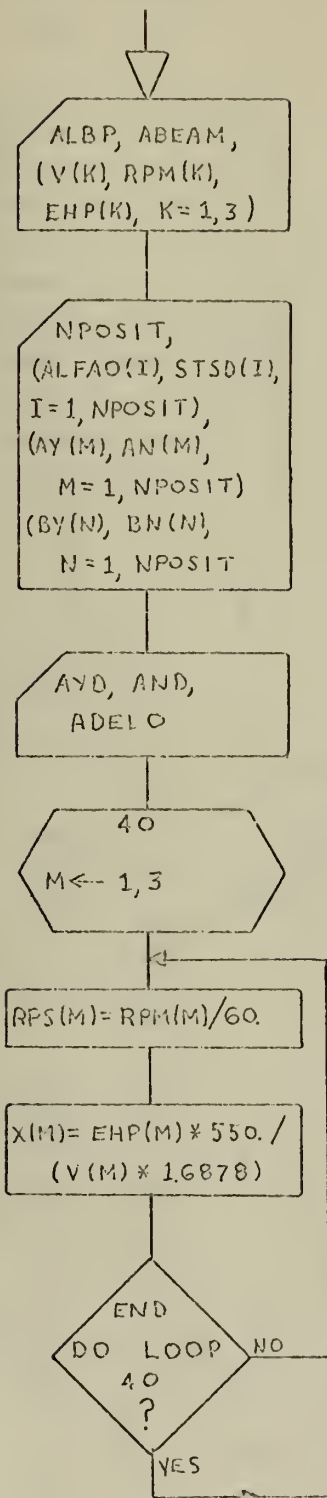
The output format for DETERM is illustrated by the sample results section of this Appendix and should be self-explanatory. In the output, X_n is represented by DX/DN , Y_n by DY/DN , etc.

III. FLOW CHART

START

PROGRAM DETERM

DETERM PG. 1



FROM DETERM
PG. 1, #1.1

#2.1

DAN = AN(L) * (0.5 *
RHO * ((V(2) * 1.6878)
** 2.) * ALBP ** 3.)
DANUP = DAN * (V(3) /
V(2)) ** 2.
SLAN = (DANUP - DAN) /
((V(3) - V(2)) * 1.6878)
ANU = SLAN / (0.5 * RHO
* V(2) * 1.6878 *
(ALBP ** 3.))

STSD(L),
ALFAO(L), ANU,
ANU

TO DETERM
PG 1, #1.1

#2.2 END
DO LOOP
41
?

YES

'FOR SHIP A
IN SHIP B
POSITION'

42

L ← 1, NPOSIT

DBY = BY(L) * (0.5 * RHO
* ((V(2) * 1.6878) ** 2.)
* ALBP ** 2.)
DBYUP = DBY * (V(3) /
V(2)) ** 2.
SLBY = (DBYUP - DBY) /
((V(3) - V(2)) * 1.6878)
BYUINT = SLBY / (0.5 *
RHO * V(2) * 1.6878 *
(ALBP ** 2.))

DBN = BN(L) * (0.5 *
RHO * ((V(2) * 1.6878)
** 2.) * ALBP ** 3.)
DSNUP = DBN * (V(3) /
V(2)) ** 2.
SLBN = (DSNUP - DBN) /
((V(3) - V(2)) * 1.6878)
BNUINT = SLBN / (0.5 *
RHO * V(2) * 1.6878 *
(ALBP ** 3.))

STSD(L),
ALFAO(L), BYUINT,
BNUINT

END
DO LOOP
42
?

YES

END


```

C READ Y FORCE AND N MOVEMENT ON SHIP A AND SIDE TO SIDE
C SEPARATION. (STSD), AND LONGITUDINAL SEPARATION (ALFAD) AT
C NPOSIT POINTS, (UP TO 20).
      READ(5,301)(ALFAD(I),STSD(I),I=1,NPOSIT)
      READ(5,304)(AV(N),AN(M),M=1,NPOSIT)
C READ Y AND N VALUES FOR SHIP A IN SHIP B POSITION.
C WITH SIGNS CHANGED.
      READ(5,204)(BY(N),BN(N),N=1,NPOSIT)
C READ DY/D(DELTA) AND DN/D(DELTA) FOR SHIP A, (DELTA
C IS RUDDER ANGLE IN RADIANS),... READ EQUILIBRIUM RUDDER
C ANGLE (ADELO) IN DEGREES FOR SHIP A IN OPEN WATER.
C
C
      READ(5,305) AYDEL,ANDEL,ADELO
C DETERMINE DY/DN FOR SHIP A (AXN)
      DO 40 M=1,3
      RPS(V)=RPM(M)/RPS
      40 X(M)=ERP(M)*EHP/(V(M)*EPS)
      AXN=((X(3)-X(1))/(RPS(3)-RPS(1)))/(0.5*RHQ*(ALBP**3.))
      1*V(2)*EPS)
C DETERMINE DY/DN FOR SHIP A (AVN)
      DAYD=AYDEL*0.5*RHQ*(ALBP**2.)*V(2)*EPS)**2.*ADELO/RAD
      DAYDU=DAYD*(RPS(3)/RPS(2))**2.
      SLAYD=(DAYDU-DAYD)/(RPS(3)-RPS(2))
      AVN=-SLAYD/(0.5*RHQ*(ALBP**3.)*V(2)*EPS)
C DETERMINE DN/DN FOR SHIP A (ANN)
      DAND=ANDEL*0.5*RHQ*(ALBP**3.)*V(2)*EPS)**2.*ADELO/RAD
      DANDU=DAND*(RPS(3)/RPS(2))**2.
      SLAND=(DANDU-DAND)/(RPS(3)-RPS(2))
      ANN=-SLAND/(0.5*RHQ*(ALBP**4.)*V(2)*EPS)
      WRITE(6,400) ALRP,AREAV,ADELO,V(2),RPM(2)
      WRITE(6,402) AXN,AVN,ANN
C DETERMINE DY/DU (AYU) AND DN/DU (ANU) FOR SHIP A AT
C VARIOUS POSITIONS.
      DO 41 L=1,NPOSIT
      DAY=AY(L)*((0.5*RHQ*(V(2)*EPS)**2.)*ALBP**2.))

```

```

PT0V00027
PT0V00028
PT0V00029
PT0V00040
PT0V00041
PT0V00042
PT0V00043
PT0V00044
PT0V00045
PT0V00046
PT0V00047
PT0V00048
PT0V00049
PT0V00050
PT0V00051
PT0V00052
PT0V00053
PT0V00054
PT0V00055
PT0V00056
PT0V00057
PT0V00058
PT0V00059
PT0V00060
PT0V00061
PT0V00062
PT0V00063
PT0V00064
PT0V00065
PT0V00066
PT0V00067
PT0V00068
PT0V00069
PT0V00070
PT0V00071
PT0V00072

```



```

DAYUP=DAY*(V(3)/V(2))**2.
SLAY=(DAYUP-DAY)/((V(3)-V(2))*EPS)
AYU=SLAY/(0.5*RHO*V(2)*EPS*(ALBP**2.))
DAN=AN(L)*{0.5*RHO*((V(2)*EPS)**2.)*ALBP**3.}
DANUP=DAN*(V(3)/V(2))**2.
SLAN=(DANUP-DAN)/((V(3)-V(2))*EPS)
ANU=SLAN/(0.5*RHO*V(2)*EPS*(ALBP**2.))
WRITE(6,401) STSD(L),ALFAQ(L),AYU,ANU
41 CONTINUE
WRITE(6,403)
DO 42 L=1,NPQST
DAY=RY(L)*{0.5*RHO*((V(2)*EPS)**2.)*ALBP**2.}
DAYUP=DAY*(V(3)/V(2))**2.
SLAY=(DAYUP-DAY)/((V(3)-V(2))*EPS)
BYUJNT=SLAY/(0.5*RHO*V(2)*EPS*(ALBP**2.))
DRN=RN(L)*{0.5*RHO*((V(2)*EPS)**2.)*ALBP**3.}
DRNUP=DRN*(V(3)/V(2))**2.
SLRN=(DRNUP-DRN)/((V(3)-V(2))*EPS)
RNUJNT=SLRN/(0.5*RHO*V(2)*EPS*(ALBP**2.))
WRITE(6,401) STSD(L),ALFAQ(L),BYUJNT,RNUJNT
42 CONTINUE
WRITE(6,900)
FORMAT('I')
STOP
END

```


V. SAMPLE RESULTS

SHIP A

LBP= 528.50 BEAM= 76.00
OPEN WATER RUDDER ANGLE= 1.20
EQUIL SPEED IN KTS= 15.00 EQUIL RPM= 65.00

DX/DN= 0.4623E-04
DY/DN= -0.5236E-05
DN/DN= 0.2615E-05

AT SIDE TO SIDE DIST= 60.00
ALFA ZERO= 0.00
DY/DU= 0.1525E-02
DN/DU= -0.6710E-03

AT SIDE TO SIDE DIST= 90.00
ALFA ZERO= 0.00
DY/DU= 0.1098E-02
DN/DU= -0.4636E-03

AT SIDE TO SIDE DIST= 70.00
ALFA ZERO= 300.00
DY/DU= 0.3359E-03
DN/DU= 0.2255E-03

AT SIDE TO SIDE DIST= 80.00
ALFA ZERO= -400.00
DY/DU= -0.4762E-03
DN/DU= 0.1728E-03

FOR SHIP A IN SHIP B POSITION

| | | |
|-----------------------|-------|--------------------|
| AT SIDE TO SIDE DIST= | 60.00 | |
| ALFA ZERO= | 0.00 | DY/DU= -0.1525E-02 |
| | | DN/DU= 0.6710E-03 |

| | | |
|-----------------------|-------|--------------------|
| AT SIDE TO SIDE DIST= | 90.00 | |
| ALFA ZERO= | 0.00 | DY/DU= -0.1098E-02 |
| | | DN/DU= 0.4636E-03 |

| | | |
|-----------------------|--------|-------------------|
| AT SIDE TO SIDE DIST= | 70.00 | |
| ALFA ZERO= | 300.00 | DY/DU= 0.3434E-03 |
| | | DN/DU= 0.8686E-04 |

| | | |
|-----------------------|---------|--------------------|
| AT SIDE TO SIDE DIST= | 80.00 | |
| ALFA ZERO= | -400.00 | DY/DU= -0.4290E-04 |
| | | DN/DU= -0.1671E-03 |

Appendix IV

PROGRAM SOLVE

- I. Discussion
- II. Input/Output Format
- III. Flow Chart
- IV. Program Listing
- V. Sample Results

Appendix IV

PROGRAM SOLVE

I. Discussion

Program SOLVE, using the hydrodynamic derivative inputs for Ship A, and for Ship A in Ship B's position, calculates the full set of derivatives for Ship B. The derivatives for both ships are dimensionalized in the program, and the applicable coefficients for the variables of the equations of motion are computed. SOLVE then calls subroutine DETER which computes the values of the coefficients of the polynomial in \mathcal{D} which is the result of evaluating the determinant of the equations of motion coefficients.

Subroutine DETER, in the process of performing necessary polynomial addition, subtraction and multiplication, calls on library routines of the IBM Scientific Subroutine Package (SSP). These subroutines are PADD, PSUB and PMPY and are fully described in the IBM Application Program publication H20-0205-3, entitled "System/360 Scientific Subroutine Package, (360A-CM-03X) Version III". These routines are included in the program listings in this appendix and were used without modification.

Once DETER has evaluated the determinant to get the characteristic polynomial in \mathcal{D} , execution is returned to the main program, SOLVE, to determine the roots of this equation, which are the stability roots for the system. The roots are determined by a fourth SSP subroutine, POLRT, used without modification and listed in this appendix.

II. Input/Output Format

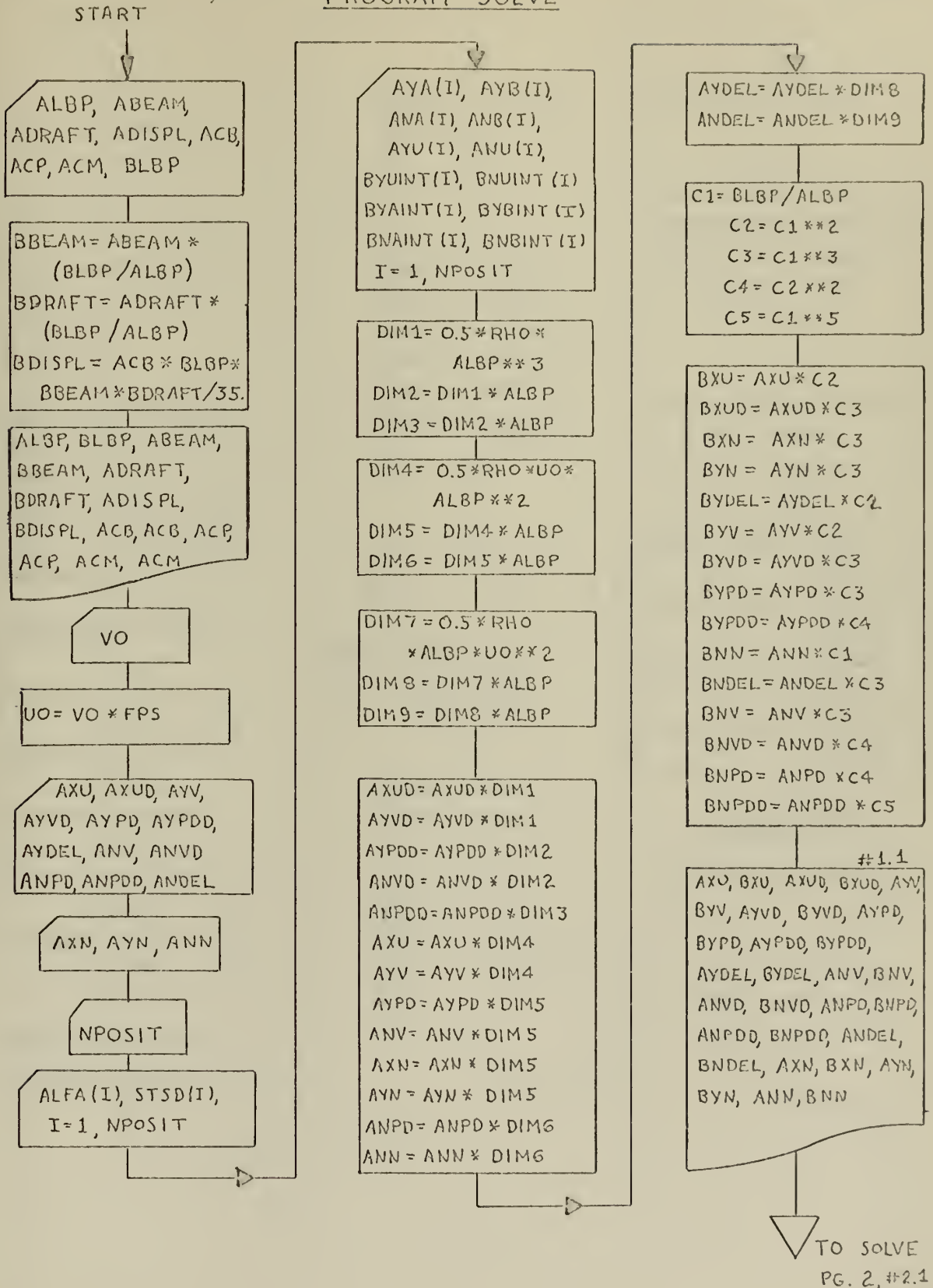
Program SOLVE's input is ordered as follows:

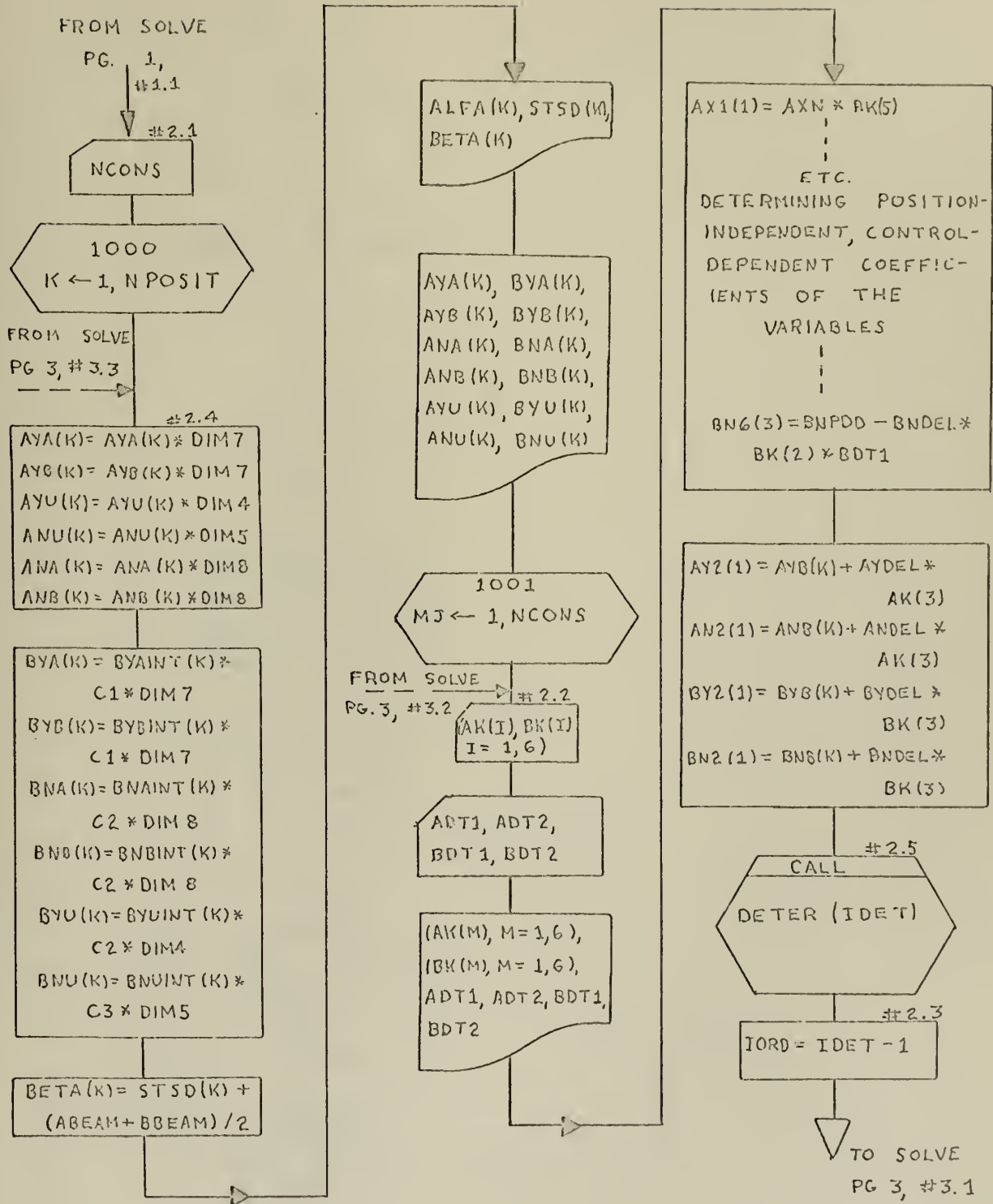
| ITEM | PROGRAM SYMBOL | CARD | COLUMNS | FORMAT |
|---|---|-------|---------|--------|
| Ship A Characteristics | ALBP, ABEAM, 1 ADRAFT, ADISPL, ACB, ACP, ACM | | 1-70 | 7F10.2 |
| Ship B Length | BLBP | 2 | 1-10 | F10.2 |
| Equil. Spd. in kts. | VO | 3 | 1-10 | F10.2 |
| Dimensionless open water derivatives, Ship A ($AXU = X_{u_A}$, etc.) | AXU, AXUD, AYU, AYVD, AYPD, AYPD, AYDEL, ANV, ANVD, ANPD, ANPOD, ANDEL | | 1-14 | E14.4 |
| | | 4-15 | | |
| Dimensionless derivatives from DETERM | AXN, AYN, ANN | 16-18 | 1-14 | E14.4 |
| # Positions examined | NPOSIT | 19 | 1-4 | I4 |
| Long'l and Lat. separation at each position | ALFA(I), STSD(I) | 20 | 1-20 | 2F10.2 |
| | (one card for each position) | | | |
| Dimensionless Y_α , Y_β , N_α , N_β , Ship A | AYA(I), AYB(I), ANA(I), ANB(I) | | 1-56 | 4E14.4 |
| | | 21 | | |
| | (one card for each position) | | | |
| Dimensionless Y_u , N_u , Ship A | AYU(I), ANU(I) | 22 | 1-28 | 2E14.4 |
| | (one card for each position) | | | |
| Dimensionless Y_u , N_u , Ship A, Ship B Position | BYUINT(I), BNUINT(I) | 23 | 1-28 | 2E14.4 |
| | (one card for each position) | | | |

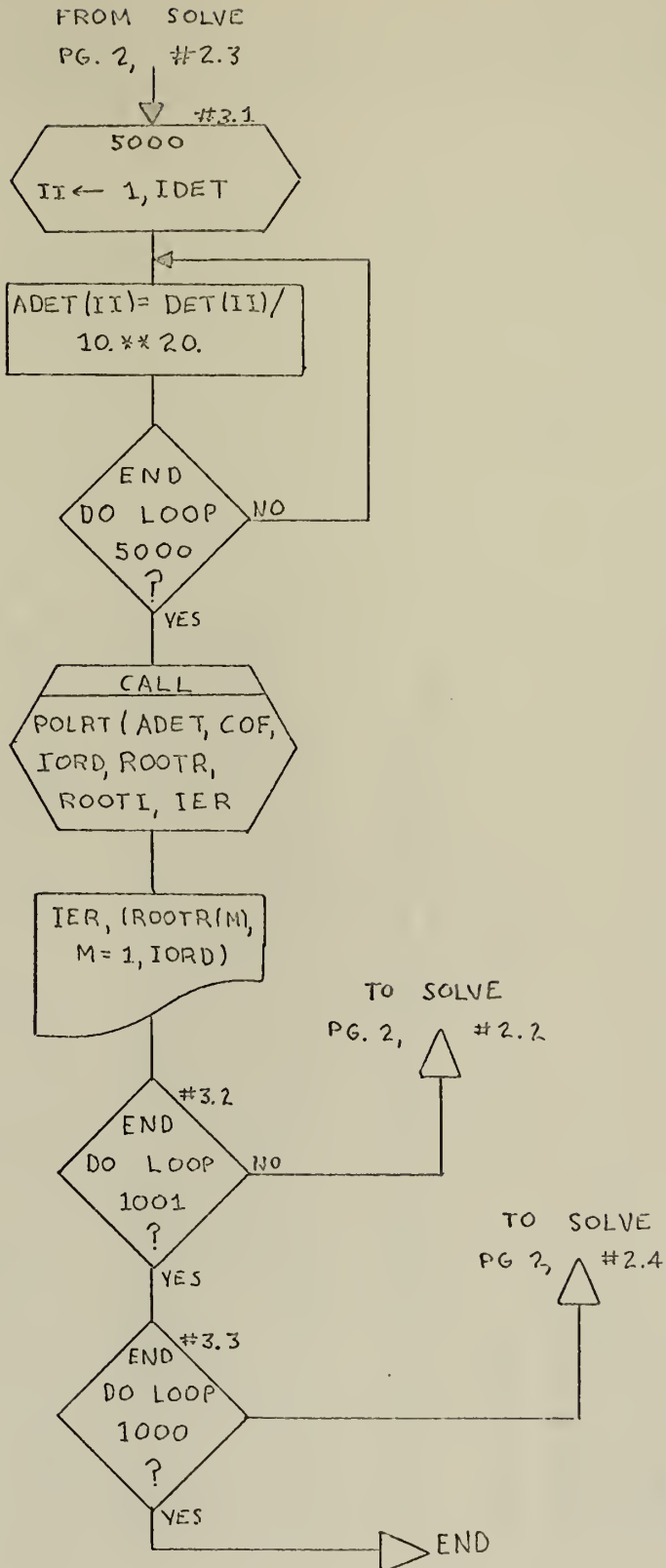
| ITEM | PROGRAM SYMBOL | CARD | COLUMNS | FORMAT |
|--|-------------------------|------|---------|--------|
| Dimensionless Y_α , Y_β , Ship A, Ship B position | BYAINT(I), BYBINT(I) | 24 | 1-28 | 2E14.4 |
| (one card for each position) | | | | |
| Dimensionless N_α , N_β , Ship A, Ship B position | BNAINT(I), BNBINT(I) | 25 | 1-28 | 2E14.4 |
| (one card for each position) | | | | |

The output format for SOLVE is illustrated by the sample results section of this Appendix. In the output, Y_α is represented by DY/DA, etc.

PROGRAM SOLVE







C TSOLVE PROGRAM LISTING

C PROGRAM SOLVE...TAKES VALUES OF HYDRODYNAMIC DERIVATIVES
C PREVIOUSLY DETERMINED FOR SHIP A AND SCALES THEM TO GET
C VALUES FOR SHIP B. THEN SUBROUTINE DETEP IS CALLED TO
C SOLVE THE DETERMINANT ARISING FROM THE SYSTEM OF EONS
C OF MOTION FOR THE TWO BODY SYSTEM. THE SOLUTION OF THE
C DETERMINANT WILL BE THE STABILITY CHARACTERISTIC EON.

```

      DIMENSION AYA(10),AYB(10),ANA(10),ANB(10),AYU(10),
1      IANU(10),RYA(10),RYB(10),BNA(10),BNB(10),BYU(10),
2      BNU(10),AK(6),BK(6),ALFA(10),STSD(10),BETA(10),AX1(3),
3      AX3(3),AY2(3),AY4(3),AY5(3),AN2(3),AN4(3),AN5(3),
4      AX3(3),BY2(3),BY4(3),BY5(3),BN2(3),BN4(3),BN5(3),
5      BY1(3),BY6(3),BN6(3),DET(24),BYUINT(10),BNUINT(10),
6      BYAINT(10),BYBINT(10),RNAINT(10),BNBINT(10),
7      ROOTIR(20),ROOTII(20),COF(20),ADET(24)

```

```

      COMMON AX1,AX3,AY2,AY4,AY5,AN2,AN4,AN5,DET,
1      RX1,RX3,RY2,RY4,RY5,BN2,BN4,BN5,BY6,BN6

```

```

      DATA FPS,RHQ /1.6787,1.0005 /

```

```

700 FORMAT(7F10.2)
701 FORMAT(F10.2)
702 FORMAT (F14.4)
703 FORMAT(I4)
704 FORMAT(4F14.4)
705 FORMAT(2F14.4)
706 FORMAT(2F10.2)
707 FORMAT(6F10.2)
708 FORMAT(4F10.2)

```

```

743 FORMAT(/15X,'CHAR. EON. REAL ROOTS ARE',/(24X,F14.4))
744 FORMAT( 15X,'OBLT TEST VALUE IEP= ',I4)
745 FORMAT( 15X,'SHIP A TIME LACS ARE',2(F10.2,2X),/

```

SOLV0001
SOLV0002
SOLV0003
SOLV0004
SOLV0005
SOLV0006
SOLV0007
SOLV0008
SOLV0009
SOLV0010
SOLV0011
SOLV0012
SOLV0013
SOLV0014
SOLV0015
SOLV0016
SOLV0017
SOLV0018
SOLV0019
SOLV0020
SOLV0021
SOLV0022
SOLV0023
SOLV0024
SOLV0025
SOLV0026
SOLV0027
SOLV0028
SOLV0029
SOLV0030
SOLV0031
SOLV0032
SOLV0033
SOLV0034
SOLV0035
SOLV0036


```

115X,'SHIP B TIME LAGS ARE',2(F10.2,3X)
746 FORMAT( 15X,'SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6' /
12(15X,3(F10.6,3X) / )
747 FORMAT(12, / ,15X,'SHIP A CONTROL SYSTEM CONSTANTS'
1, 1 TO 6' / 2(15X,3(F10.6,3X) / )
748 FORMAT( / ,20X,'SHIP A',15X,'SHIP B', /
115X,'DY/DA',5X,F14.4,7X,F14.4 /
215X,'DY/DB',5X,F14.4,7X,F14.4 /
315X,'DN/DA',5X,F14.4,7X,F14.4 /
415X,'DN/DB',5X,F14.4,7X,F14.4 /
515X,'DY/DU',5X,F14.4,7X,F14.4 /
615X,'DN/DU',5X,F14.4,7X,F14.4 )

C
749 FORMAT( / ,15X,'AT ALFA= ',F7.1,' STD= ',F7.1,
1, AND RETA= ',F7.1/15X,' THE POSITION DEPENDENT DER
2,IVATIVES ARE' )

C
750 FORMAT( / / / / ,26X,'SHIP A',15X,'SHIP B' /
115X,'LRP= ', 7X,F10.2,11X,F10.2 /
215X,'BFAM= ', 6X,F10.2,11X,F10.2 /
315X,'DRAFT= ', 5X,F10.2,11X,F10.2 /
415X,'DISPL= ', 5X,F10.2,11X,F10.2 /
515X,'CR= ', 8X,F10.4,11X,F10.4 /
615X,'CP= ', 8X,F10.4,11X,F10.4 /
715X,'CM= ', 8X,F10.4,11X,F10.4 )

C
751 FORMAT( / / / / ,26X,'SHIP A',15X,'SHIP B' / 12X,'DY/DU',5X,
1F14.4,7X,F14.4/12X,'DX/DUDDT',2X,F14.4,7X,F14.4/12X,
2'DY/DV',5X,F14.4,7X,F14.4/12X,'DY/DVDDT',2X,F14.4,7X,F
314.4/12X,'DY/DPODDT',3X,F14.4,7X,F14.4/12X,'DY/DPODDT',
41X,F14.4,7X,F14.4/12X,'DY/DDEL',3X,F14.4,7X,F14.4 /
512X,'DN/DV',5X,F14.4,7X,F14.4/12X,'DN/DVDDT',2X,F14.4,
67X,F14.4/12X,'DN/DPODDT',2X,F14.4,7X,F14.4/12X,
7'DN/DPODDT',1X,F14.4,7X,F14.4/12X,'DN/DDEL',3X,F14.4,
87X,F14.4/12X,'DX/DN',5X,F14.4,7X,F14.4/12X,'DY/DN',5X
9,F14.4,7X,F14.4/12X,'DN/DN',5X,F14.4,7X,F14.4)

```

```

SOLV0027
SOLV0033
SOLV0039
SOLV0040
SOLV0041
SOLV0042
SOLV0043
SOLV0044
SOLV0045
SOLV0046
SOLV0047
SOLV0048
SOLV0049
SOLV0050
SOLV0051
SOLV0052
SOLV0053
SOLV0054
SOLV0055
SOLV0056
SOLV0057
SOLV0058
SOLV0059
SOLV0060
SOLV0061
SOLV0062
SOLV0063
SOLV0064
SOLV0065
SOLV0066
SOLV0067
SOLV0068
SOLV0069
SOLV0070
SOLV0071
SOLV0072

```



```

C      752 FORMAT( //,15X,' THE DETERMINANT IN ASCENDING D' /
115X,' POWERS IS',/(24X,E14.4))
C
C      READ(5,700)ALBP,ABEAM,ADRAFT,ADISPL,ACB,ACP,ACM
C      READ(5,701)BLRP
C      COMPUTE SHIP B CHARACTERISTICS
C      BBEAM=ABEAM*BLRP/ALBP
C      BDRAFT=ADRAFT*BBEAM/ABEAM
C      BDISPL=ACB*BLRP*BBEAM*BDRAFT/35.
C      WRITE(6,750)ALBP,BLRP,ABEAM,BBEAM,ADRAFT,BDRAFT,BDISPL
C      1,BDISPL,ACB,ACB,ACP,ACP,ACM,ACM
C      VO IS EQUILIBRIUM SPEED IN KTS. UC IN FT/SEC
C      READ(5,701) VN
C      UD=VN*FPS
C
C      READ SHIP A POSITION-INDEPENDENT DERIVATIVES, USING
C      STROM-TEJSEN VALUES IN NON-DIMENSIONAL FORM,
C      READ(5,702)AXU,AXUD,AYV,AYVD,AYPD,AYDEL,ANV,ANVD
C      1,AND,ANPDD,ANDEL
C      THESE DERIVATIVES FROM PROGRAM DETERM ARE ALSO POSITION-
C      INDEPENDENT AND DIMENSIONLESS
C      READ(5,702) AXN,AYN,ANN
C
C      READ POSITION-DEPENDENT DERIVATIVES IN NON-DIMENSIONAL
C      FORM, AVA,AVB,ANA,ANB ARE FROM PROGRAM INTERC.
C      AYU,ANU,BYUINT AND BNUINT FROM PROGRAM DETERM
C      BYAINT,BYRINT,BNAINT AND BNRINT FROM PROGRAM INTERC. RV
C      ASSUMING SHIP A IN SHIP B POSITION.
C      NPOSIT IS THE NO. OF POSITIONS OF INTEREST (UP TO 10)
C
C      READ(5,703)NPOSIT
C      READ(5,706)(ALFA(I),STSD(I),I=1,NPOSIT)
C      READ(5,704)(AVA(I),AVB(I),ANA(I),ANB(I),I=1,NPOSIT)
C      READ(5,705)(AYU(I),ANU(I),I=1,NPOSIT)
C      READ(5,705)(BYUINT(I),BNUINT(I),I=1,NPOSIT)

```



```

C1=BLBP/ALBP
C2=C1**2,
C3=C1**3,
C4=C2**2,
C5=C1**5.

DETERMINING SHIP B'S POSITION-INDEPENDENT DERIVATIVES.
IN DIMENSIONAL FORM

BXU =AXU *C2
BXUD =AXUD *C3
BXN =AXN *C3
BYN =AYN *C3
BYDEL=AYDEL*C2
BYV =AYV *C2
BYVD =AYVD *C3
BYPO =AYPO *C3
BYPDD=AYPDD*C4
BYN =ANN *C1
BYDEL=AYDEL*C3
BYV =ANV *C3
BYVD =ANVD *C4
BYPO =ANPO *C4
BYPDD=ANPDD*C5
WRITE(6,751)AXU,BXU,AXUD,BXUD,AYV,BYV,AYVD,BYVD,
1AYPO,AYPDD,BYPDD,AYDEL,BYDEL,ANV,BNV,ANVD,BNVD,ANPD,
2BNPD,ANPDD,BNPDD,ANDEL,AXN,BXN,AYN,BYN,ANN,BNN
NCONS IS THE NUMBER OF SETS OF CONTROL SYSTEM CONSTANTS
TO BE USED. ANY NUMBER OF SETS OF CONSTANTS, SIX PER
SHIP, MAY BE USED

READ(5,703) NCONS
DO LOOP CYCLES CALCULATION OF CONTROL-DEPENDENT PAR-

```



```

C AMETERS ONCE FOR EACH SET OF CONTROL SYSTEM CONSTANTS
C BEING EXAMINED AT EACH POSITION.
C
C      DO 1000 K=1,NPCST
C
C DIMENSIONALIZE THE POSITION-DEPENDENT DERIVATIVES FOR
C SHIP A
      AYA(K)=AYA(K)*DIM7
      AYB(K)=AYB(K)*DIM7
      AYU(K)=AYU(K)*DIM4
      ANU(K)=ANU(K)*DIM5
      ANA(K)=ANA(K)*DIM8
      ANR(K)=ANR(K)*DIM9
C EVALUATE POSITION-DEPENDENT DERIVATIVES FOR SHIP B
      BYA(K)=BYAINT(K)*CI*DIMP7
      BYB(K)=-BYRINT(K)*CI*DIMP7
      BNA(K)=BNAINT(K)*C2 *DIMP8
      BNB(K)=-BNRINT(K)*C2*DIMP8
      BVU(K)=BVUINT(K)*C2*DIMP4
      BNU(K)=BNUINT(K)*C3*DIMP5
C
C WRITE THE POSITION DEPENDENT DERIVATIVES FOR BOTH SHIPS.
      PETA(K)=STSD(K)+(AREAM+BBEAM)/2.
      WRITE(6,746)ALEA(K),STSD(K),PETA(K)
      WRITE(6,748)AYA(K),BYA(K),AYB(K),BYB(K),ANA(K),BNA(K),
      1ANR(K),BNB(K),AYU(K),BVU(K),ANU(K),BNU(K)
      DO 1001 MJ=1,NCONS
C
C READ CONTROL SYSTEM CONSTANTS AND TIME LAGS
C
C      READ(5,707)(AK(I),I=1,6)
C      READ(5,707)(BK(I),I=1,6)
C      READ(5,708)ACT1,ADT2,BDT1,BDT2
C WRITE CONTROL SYSTEM CONSTANTS AND TIME LAGS
      WRITE(6,747)(AK(M),M=1,6)
      WRITE(6,746)(BK(M),M=1,6)

```

```

SOI.V0181
SOI.V0182
SOI.V0183
SOI.V0184
SOI.V0185
SOI.V0186
SOI.V0187
SOI.V0188
SOI.V0189
SOI.V0190
SOI.V0191
SOI.V0192
SOI.V0193
SOI.V0194
SOI.V0195
SOI.V0196
SOI.V0197
SOI.V0198
SOI.V0199
SOI.V0200
SOI.V0201
SOI.V0202
SOI.V0203
SOI.V0204
SOI.V0205
SOI.V0206
SOI.V0207
SOI.V0208
SOI.V0209
SOI.V0210
SOI.V0211
SOI.V0212
SOI.V0213
SOI.V0214
SOI.V0215
SOI.V0216

```



```

WRITE(6,745)ADT1,ADT2,BDT1,BDT2
C
C DETERMINING POSITION-INDEPENDENT BUT CONTROL-DEPENDENT
C COEFFICIENTS OF THE VARIABLES.
      AX1(1)=AXN*AK(5)
      AX1(2)=AXN*(AK(6)-AK(5)*ADT2)
      AX1(3)=-AXN*AK(6)*ADT2
      AX3(1)=AXU
      AX3(2)=AXUD
      AY2(2)=AYDEL*(AK(4)-AK(3)*ADT1)
      AY2(3)=-AYDEL*AK(4)*ADT1
      AY4(1)=AYV
      AY4(2)=AYVD
      AY5(1)=AYDEL*AK(1)
      AY5(2)=AYPD+AYDEL*(AK(2)-AK(1)*ADT1)
      AY5(3)=AYDD-AYDEL*AK(2)*ADT1
      AN2(2)=ANDEL*(AK(4)-AK(3)*ADT1)
      AN2(3)=-ANDEL*AK(4)*ADT1
      AN4(1)=ANV
      AN4(2)=ANVD
      AN5(1)=ANDEL*AK(1)
      AN5(2)=ANPD+ANDEL*(AK(2)-AK(1)*ADT1)
      AN5(3)=ANPD-ANDEL*AK(2)*ADT1
C
C SHIP R
      BX1(1)=BXN*BK(5)
      BX1(2)=BXN*(BK(6)-BK(5)*BDT2)+BYU
      BX1(3)= BXUD-BXN*BK(6)*BDT2
      BX3(1)=BXU
      BX3(2)=BXUD
      BY2(2)=BYDEL*(BK(4)-BK(3)*BDT1)+BYV
      BY2(3)=BYVD-BYDEL*BK(4)*BDT1
      BY4(1)=BYV
      BY4(2)=BYVD
      BY5(1)=BYV*UD
      BY5(2)=BYVD*UD

```

```

CBLV0017
CBLV0018
CBLV0019
CBLV0020
CBLV0021
CBLV0022
CBLV0023
CBLV0024
CBLV0025
CBLV0026
CBLV0027
CBLV0028
CBLV0029
CBLV0030
CBLV0031
CBLV0032
CBLV0033
CBLV0034
CBLV0035
CBLV0036
CBLV0037
CBLV0038
CBLV0039
CBLV0040
CBLV0041
CBLV0042
CBLV0043
CBLV0044
CBLV0045
CBLV0046
CBLV0047
CBLV0048
CBLV0049
CBLV0050
CBLV0051
CBLV0052

```



```

RY6(1)=RYDEL*RK(1)-RYV*UN
RY6(2)=RYPD+RYDEL*(RK(2)-BK(1)*BDT1)-RYVD*UN
RY6(3)=RYPPD-RYDEL*RK(2)*BDT1
RN2(2)=RNDEL*(RK(4)-BK(3)*BDT1)+RNV
RN2(3)=RNVD-RNDEL*RK(4)*BDT1
RN4(1)=RNV
RN4(2)=RNVD
RN5(1)=RNVD*UN
RN5(2)=RNVD*UN
RN6(1)=RNDEL*RK(1)-RNVD*UN
RN6(2)=RNPD+RNDEL*(RK(2)-BK(1)*BDT1)-RNVD*UN
RN6(3)=RNPPD-RNDEL*RK(2)*BDT1

```

```

C
C DETERMINE POSITION- AND CONTROL-DEPENDENT COEFFICIENTS
C OF THE VARIABLES
C

```

```

AY2(1)=AYR(K)+AYDEL*AK(3)
AN2(1)=ANR(K)+ANDEL*AK(3)
RY2(1)=RYR(K)+RYDEL*RK(3)
RN2(1)=RNR(K)+RNDEL*RK(3)

```

```

C
CALL DETER (IDET)
IORD=IDET-1
DO 5000 II=1,IDET
  ADET(II)=DET(II)/10.**20
5000 CONTINUE
CALL POLRT(ADET,COF,IORD,POCTR,ROOT1,IFR)

```

```

C
C POLRT IS AN IBM SCIENTIFIC SUBROUTINE PACKAGE, (SSP).
C ROUTINE WHICH, WHEN CALLED WITH THE
C VECTOR OF COEFFICIENTS OF A POLYNOMIAL, ARRANGED FROM
C LOWEST TO HIGHEST ORDER, RETURNS THE REAL (ROOTR) AND
C IMAGINARY (ROOTI) ROOTS OF THE POLYNOMIAL. COF IS A DUMMY
C VECTOR, IFR IS A TEST VALUE WHICH EQUALS 7500 WHEN NO
C DISCREPANCIES EXIST.
C

```

```

SOLV0250
SOLV0254
SOLV0255
SOLV0256
SOLV0257
SOLV0258
SOLV0259
SOLV0260
SOLV0261
SOLV0262
SOLV0263
SOLV0264
SOLV0265
SOLV0266
SOLV0267
SOLV0268
SOLV0269
SOLV0270
SOLV0271
SOLV0272
SOLV0273
SOLV0274
SOLV0275
SOLV0276
SOLV0277
SOLV0278
SOLV0279
SOLV0280
SOLV0281
SOLV0282
SOLV0283
SOLV0284
SOLV0285
SOLV0286
SOLV0287
SOLV0288

```



```

WRITE(6,744)IER
WRITE(6,742)(ROOTR(M),M=1,ICRD)
1001 CONTINUE
1000 CONTINUE
      STOP
      END

```

```

SOLV0000
SOLV0000
SOLV0001
SOLV0020
SOLV0002
SOLV0004

```



```

SUBROUTINE DETER (IDET)
    DIMENSION AX1(3),AX3(3),AY2(3),AY4(3),AY5(3),AN2(3),
    1AN4(3),AN5(3),BX1(3),BX3(3),BY2(3),BY5(3),BN2(3),
    2BN4(3),BN5(3),BY6(3),BN6(3),BY4(3),
    3AX1BY3(6),AX3BY1(6),FAC(12),EFAC3(24),
    4BN6BY5(6),BY6BN5(6),FAC1(12),SUM1(24),
    5BN6BY2(6),BY6BN2(6),FAC3(12),SUM(24),
    6BN6BY4(6),BY6BN4(6),FAC5(12),DET(24),
    7AY4AN2(6),AN4AY2(6),FAC2(12),
    8AY5AN4(6),AN5AY4(6),FAC4(12),
    9AY2AN5(6),AN2AY5(6),FAC6(12),EFAC1(24),EFAC2(24)
    COMMON AX1,AX3,AY2,AY4,AY5,AN2,AN4,AN5,DET,
    1    BX1,BX3,BY2,BY4,BY5,BN2,BN4,BN5,BY6,BN6
    EVALUATING THE DETERMINANT
    DETERMINANT (DET)=(AX1*BX3-BX1*AX3)
    *((BN6*BY5-BY6*BN5)*(AY4*AN2-AN4*AY2)
    +(BN6*BY2-BY6*BN2)*(AY5*AN4-AN5*AY4)
    +(BN6*BY4-BY6*BN4)*(AY2*AN5-AN2*AY5))
    THIS IS SHORTENED TO SYMBOLS AS
    DET=FAC*(FAC1*FAC2+FAC3*FAC4+FAC5*FAC6)
    EVALUATING FAC
    CALL PMPY(AX1,BX3,IRES1,AX1,3,BX3,2)
    PMPY IS AN IBM SCIENTIFIC SUBROUTINE PACKAGE (SSP) SUB-
    ROUTINE WHICH IS CALLED BY CALL PMPY (7,IDIM7,X,IDIMX,Y,
    IDIMY) AND MULTIPLIES TWO POLYNOMIALS.,Z=X*Y. IDIMX,
    IDIMY AND IDIMZ ARE THE DIMENSIONS OF THE RESPECTIVE
    POLYNOMIALS, WHERE DIMENSION = DEGREE+1.
    CALL PMPY(AX3,BX1,IRES2,AX3,2,BX1,3)

```

```

DET00001
DET00002
DET00003
DET00004
DET00005
DET00006
DET00007
DET00008
DET00009
DET00010
DET00011
DET00012
DET00013
DET00014
DET00015
DET00016
DET00017
DET00018
DET00019
DET00020
DET00021
DET00022
DET00023
DET00024
DET00025
DET00026
DET00027
DET00028
DET00029
DET00030
DET00031
DET00032
DET00033
DET00034
DET00035
DET00036

```



```

C PSUR IS AN SSP SUBROUTINE THAT SUBTRACTS ONE POLYNOMIAL
C FROM ANOTHER. CALL PSUB(7, IDIMZ, X, IDIMX, Y, IDIMY)
C 7=X-Y
C CALL PSUR(FAC, IFAC, AX1BX3, IRES1, AX3BX1, IRES2)
C EVALUATING FAC1
C CALL PMPY(PN6BY5, IPFS3, PN6, 3, BY5, 2)
C CALL PMPY(BY6BN5, IRES4, BY6, 3, BN5, 2)
C CALL PSUR(FAC1, IFAC1, BN6BY5, IRES3, BY6BN5, IRES4)
C MFAC=1
C EVALUATING FAC2
C CALL PMPY(AY4AN2, IRES5, AY4, 2, AN2, 3)
C CALL PMPY(AN4AY2, IRES6, AN4, 2, AY2, 3)
C CALL PSUR(FAC2, IFAC2, AY4AN2, IRES5, AN4AY2, IRES6)
C MFAC=MFAC+1
C EVALUATING FAC3
C CALL PMPY(BN6BY2, IRES7, BN6, 3, BY2, 3)
C CALL PMPY(BY6BN2, IRES8, BY6, 3, BN2, 3)
C CALL PSUR(FAC3, IFAC3, BN6BY2, IRES7, BY6BN2, IRES8)
C MFAC=MFAC+1
C EVALUATING FAC4
C CALL PMPY(AY5AN4, IRES9, AY5, 3, AN4, 2)
C CALL PMPY(AN5AY4, IRES10, AN5, 3, AY4, 2)
C CALL PSUR(FAC4, IFAC4, AY5AN4, IRES9, AN5AY4, IRES10)
C MFAC=MFAC+1
C EVALUATING FAC5
C CALL PMPY(BN6BY4, IRES11, BN6, 3, BY4, 2)
C CALL PMPY(BY6BN4, IRES12, BY6, 3, BN4, 2)
C CALL PSUR(FAC5, IFAC5, BN6BY4, IRES11, BY6BN4, IRES12)
C MFAC=MFAC+1
C EVALUATING FAC6
C CALL PMPY(AY2AN5, IRES13, AY2, 3, AN5, 3)
C CALL PMPY(AN2AY5, IRES14, AN2, 3, AY5, 3)
C CALL PSUR(FAC6, IFAC6, AY2AN5, IRES13, AN2AY5, IRES14)
C MFAC=MFAC+1

```

```

DET00027
DET00028
DET00029
DET00030
DET00031
DET00032
DET00033
DET00034
DET00035
DET00036
DET00037
DET00038
DET00039
DET00040
DET00041
DET00042
DET00043
DET00044
DET00045
DET00046
DET00047
DET00048
DET00049
DET00050
DET00051
DET00052
DET00053
DET00054
DET00055
DET00056
DET00057
DET00058
DET00059
DET00060
DET00061
DET00062
DET00063
DET00064
DET00065
DET00066
DET00067
DET00068
DET00069
DET00070
DET00071
DET00072

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```

C      EVALUATING FAC1*FAC2, FAC3*FAC4, AND FAC5*FAC6
      CALL DMPV(FFAC1, IRES15, FAC1, IFAC1, FAC2, IFAC2)
      CALL DMPV(FFAC2, IRES16, FAC3, IFAC3, FAC4, IFAC4)
      CALL DMPV(FFAC3, IRES17, FAC5, IFAC5, FAC6, IFAC6)

C
C      EVALUATING THE SUMS
C      PADD IS AN SSP SUBROUTINE THAT ADDS 2 POLYNOMIALS
C      CALL PADD(Z, IDIM7, X, IDIMX, Y, IDIMY)
C      Z=X+Y
C
      CALL PADD(SUM1, ISUM1, FFAC1, IRES15, FFAC2, IRES16)
      CALL PADD(SUM, ISUM, SUM1, ISUM1, FFAC3, IRES17)
C      MULTIPLY FAC*SUM. TO GET DETERMINANT. DET
      CALL DMPV(DET, IDET, FAC, IFAC, SUM, ISUM)
      RETURN
      END

```

```

DET00073
DET00074
DET00075
DET00076
DET00077
DET00078
DET00079
DET00080
DET00081
DET00082
DET00083
DET00084
DET00085
DET00086
DET00087
DET00088

```



```

C      SUBROUTINE PSUB(Z, IDIMZ, X, IDIMX, Y, IDIMY)
C      THE ABOVE CARD SHOULD BE PLACED IN PROPER SEQUENCE
C      BEFORE COMPILING THIS UNDER IBM FORTRAN G.
C
C      .....
C      SUBROUTINE PSUB
C      PURPOSE
C      SUBTRACT ONE POLYNOMIAL FROM ANOTHER
C
C      USAGE
C      CALL PSUB(Z, IDIMZ, X, IDIMX, Y, IDIMY)
C
C      DESCRIPTION OF PARAMETERS
C      Z      - VECTOR OF RESULTANT COEFFICIENTS, ORDERED FROM
C              SMALLEST TO LARGEST POWER
C      IDIMZ  - DIMENSION OF Z (CALCULATED)
C      X      - VECTOR OF COEFFICIENTS FOR FIRST POLYNOMIAL, ORDERED FROM
C              SMALLEST TO LARGEST POWER
C      IDIMX  - DIMENSION OF X (DEGREE IS IDIMX-1)
C      Y      - VECTOR OF COEFFICIENTS FOR SECOND POLYNOMIAL,
C              ORDERED FROM SMALLEST TO LARGEST POWER
C      IDIMY  - DIMENSION OF Y (DEGREE IS IDIMY-1)
C
C      REMARKS
C      VECTOR Z MAY BE IN SAME LOCATION AS EITHER VECTOR X OR
C      VECTOR Y ONLY IF THE DIMENSION OF THAT VECTOR IS NOT LESS
C      THAN THE OTHER INPUT VECTOR
C      THE RESULTANT POLYNOMIAL MAY HAVE TRAILING ZERO COEFFICIENTS
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C      NONE
C
C      METHOD
C      DIMENSION OF RESULTANT VECTOR IDIMZ IS CALCULATED AS THE

```



```

C          LARGER OF THE TWO INPUT VECTOR DIMENSIONS. COEFFICIENTS IN
C          VECTOR Y ARE THEN SUBTRACTED FROM CORRESPONDING COEFFICIENTS
C          IN VECTOR X.
C          .....
C          DIMENSION Z(1),X(1),Y(1)
C
C          TEST DIMENSIONS OF SUMMANDS
C
C          NDIM=IDIMX
C          IF (IDIMX-IDIMY) 10,20,20
C          10 NDIM=IDIMY
C          20 IF (NDIM) 90,90,30
C          30 DO 80 I=1,NDIM
C             IF (1-IDIM*X) 40,40,60
C             40 IF (1-IDIM*Y) 50,50,70
C             50 Z(I)=X(I)-Y(I)
C             GO TO 80
C             60 Z(I)=-Y(I)
C             GO TO 80
C             70 Z(I)=X(I)
C             80 CONTINUE
C             90 IDIM7=NDIM
C             RETURN
C             END
C          .....
C          PSUB 340 PSUB00037
C          PSUB 350 PSUB00038
C          PSUB 360 PSUB00039
C          PSUB 370 PSUB00040
C          PSUB 380 PSUB00041
C          PSUB 390 PSUB00042
C          PSUB 410 PSUB00043
C          PSUB 420 PSUB00044
C          PSUB 420 PSUB00045
C          PSUB 440 PSUB00046
C          PSUB 450 PSUB00047
C          PSUB 460 PSUB00048
C          PSUB 470 PSUB00049
C          PSUB 480 PSUB00050
C          PSUB 490 PSUB00051
C          PSUB 500 PSUB00052
C          PSUB 510 PSUB00053
C          PSUB 520 PSUB00054
C          PSUB 530 PSUB00055
C          PSUB 540 PSUB00056
C          PSUB 550 PSUB00057
C          PSUB 560 PSUB00058
C          PSUB 570 PSUB00059
C          PSUB 580 PSUB00060
C          PSUB 590 PSUB00061
C          PSUB 600 PSUB00062

```


| | | | | |
|---|--|------|-----|-----------|
| C | SUBROUTINE PADD(Z, IDIM7, X, IDIMX, Y, IDIMY) | PADD | 397 | PADD00001 |
| C | THE ABOVE CARD SHOULD BE PLACED IN PROPER SEQUENCE | | | PADD00002 |
| C | BEFORE COMPILING THIS UNDER IBM FORTRAN G. | | | PADD00003 |
| C | | PADD | 10 | PADD00004 |
| C | | PADD | 20 | PADD00005 |
| C | | PADD | 30 | PADD00006 |
| C | | PADD | 40 | PADD00007 |
| C | | PADD | 50 | PADD00008 |
| C | | PADD | 60 | PADD00009 |
| C | | PADD | 70 | PADD00010 |
| C | | PADD | 80 | PADD00011 |
| C | | PADD | 90 | PADD00012 |
| C | | PADD | 100 | PADD00013 |
| C | | PADD | 110 | PADD00014 |
| C | | PADD | 120 | PADD00015 |
| C | | PADD | 130 | PADD00016 |
| C | | PADD | 140 | PADD00017 |
| C | | PADD | 150 | PADD00018 |
| C | | PADD | 160 | PADD00019 |
| C | | PADD | 170 | PADD00020 |
| C | | PADD | 180 | PADD00021 |
| C | | PADD | 190 | PADD00022 |
| C | | PADD | 200 | PADD00023 |
| C | | PADD | 210 | PADD00024 |
| C | | PADD | 220 | PADD00025 |
| C | | PADD | 230 | PADD00026 |
| C | | PADD | 240 | PADD00027 |
| C | | PADD | 250 | PADD00028 |
| C | | PADD | 260 | PADD00029 |
| C | | PADD | 270 | PADD00030 |
| C | | PADD | 280 | PADD00031 |
| C | | PADD | 290 | PADD00032 |
| C | | PADD | 300 | PADD00033 |
| C | | PADD | 310 | PADD00034 |
| C | | PADD | 320 | PADD00035 |
| C | | PADD | 330 | PADD00036 |

SUBROUTINE PADD(Z, IDIM7, X, IDIMX, Y, IDIMY)
 THE ABOVE CARD SHOULD BE PLACED IN PROPER SEQUENCE
 BEFORE COMPILING THIS UNDER IBM FORTRAN G.

 SUBROUTINE PADD

 PURPOSE
 ADD TWO POLYNOMIALS

 USAGE
 CALL PADD(Z, IDIM7, X, IDIMX, Y, IDIMY)

 DESCRIPTION OF PARAMETERS
 Z - VECTOR OF RESULTANT COEFFICIENTS, ORDERED FROM
 SMALLEST TO LARGEST POWER
 IDIM7 - DIMENSION OF Z (CALCULATED)
 X - VECTOR OF COEFFICIENTS FOR FIRST POLYNOMIAL, ORDERED FROM
 SMALLEST TO LARGEST POWER
 IDIMX - DIMENSION OF X (DEGREE IS IDIMX-1)
 Y - VECTOR OF COEFFICIENTS FOR SECOND POLYNOMIAL,
 ORDERED FROM SMALLEST TO LARGEST POWER
 IDIMY - DIMENSION OF Y (DEGREE IS IDIMY-1)

 REMARKS
 VECTOR Z MAY BE IN SAME LOCATION AS EITHER VECTOR X OR
 VECTOR Y ONLY IF THE DIMENSION OF THAT VECTOR IS NOT LESS
 THAN THE OTHER INPUT VECTOR
 THE RESULTANT POLYNOMIAL MAY HAVE TRAILING ZERO COEFFICIENTS

 SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
 NONE

 METHOD
 DIMENSION OF RESULTANT VECTOR IDIM7 IS CALCULATED AS THE


```

C          LARGER OF THE TWO INPUT VECTOR DIMENSIONS. CORRESPONDING
C          COEFFICIENTS ARE THEN ADDED TO FORM 7.
C          .....
C          DIMENSION 7(1),X(1),Y(1)
C
C          TEST DIMENSIONS OF SUMMANDS
C
C          NDIM=IDIMX
C          IF (IDIMX-IDIMY) 10,20,20
C
C          10 NDIM=IDIMY
C          20 IF(NDIM) 90,90,30
C          30 DO 80 I=1,NDIM
C             IF(I-IDIMX) 40,40,60
C             IF(I-IDIMY) 50,50,70
C             Z(I)=X(I)+Y(I)
C             GO TO 80
C          40 Z(I)=Y(I)
C          50 GO TO 80
C          60 Z(I)=X(I)
C          70 Z(I)=X(I)
C          80 CONTINUE
C          90 IDIM7=NDIM
C          RETURN
C          END
C          .....
C          PA00 340 PA000037
C          PA00 350 PA000038
C          PA00 360 PA000039
C          PA00 370 PA000040
C          PA00 380 PA000041
C          PA00 400 PA000042
C          PA00 410 PA000043
C          PA00 420 PA000044
C          PA00 430 PA000045
C          PA00 440 PA000046
C          PA00 450 PA000047
C          PA00 460 PA000048
C          PA00 470 PA000049
C          PA00 480 PA000050
C          PA00 490 PA000051
C          PA00 500 PA000052
C          PA00 510 PA000053
C          PA00 520 PA000054
C          PA00 530 PA000055
C          PA00 540 PA000056
C          PA00 550 PA000057
C          PA00 560 PA000058
C          PA00 570 PA000059
C          PA00 580 PA000060
C          PA00 590 PA000061

```



```

SUBROUTINE PMPY(Z, IDIM7, X, IDIMX, Y, IDIMY)
  THE ABOVE CARD SHOULD BE PLACED IN PROPER SEQUENCE
  BEFORE COMPILING THIS UNDER IBM FORTRAN G.

.....

SUBROUTINE PMPY
  PURPOSE
    MULTIPLY TWO POLYNOMIALS

  USAGE
    CALL PMPY(7, IDIM7, X, IDIMX, Y, IDIMY)

  DESCRIPTION OF PARAMETERS
    7 - VECTOR OF RESULTANT COEFFICIENTS, ORDERED FROM
        SMALLEST TO LARGEST POWER
    IDIMZ - DIMENSION OF 7 (CALCULATED)
    X - VECTOR OF COEFFICIENTS FOR FIRST POLYNOMIAL, ORDERED FROM
        SMALLEST TO LARGEST POWER
    IDIMX - DIMENSION OF X (DEGREE IS IDIMX-1)
    Y - VECTOR OF COEFFICIENTS FOR SECOND POLYNOMIAL,
        ORDERED FROM SMALLEST TO LARGEST POWER
    IDIMY - DIMENSION OF Y (DEGREE IS IDIMY-1)

  REMARKS
    Z CANNOT BE IN THE SAME LOCATION AS X
    Y CANNOT BE IN THE SAME LOCATION AS Y

  SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
    NONE

  METHOD
    DIMENSION OF Z IS CALCULATED AS IDIMX+IDIMY-1
    THE COEFFICIENTS OF 7 ARE CALCULATED AS SUM OF PRODUCTS
    OF COEFFICIENTS OF X AND Y, WHOSE EXPONENTS ADD UP TO THE

```

```

PMPY 300 PMPY0001
PMPY0002
PMPY0003
PMPY 10 PMPY0004
PMPY 20 PMPY0005
PMPY 30 PMPY0006
PMPY 40 PMPY0007
PMPY 50 PMPY0008
PMPY 60 PMPY0009
PMPY 70 PMPY0010
PMPY 80 PMPY0011
PMPY 90 PMPY0012
PMPY 100 PMPY0013
PMPY 110 PMPY0014
PMPY 120 PMPY0015
PMPY 130 PMPY0016
PMPY 140 PMPY0017
PMPY 150 PMPY0018
PMPY0019
PMPY0020
PMPY 160 PMPY0021
PMPY 170 PMPY0022
PMPY 180 PMPY0023
PMPY 190 PMPY0024
PMPY 200 PMPY0025
PMPY 210 PMPY0026
PMPY 220 PMPY0027
PMPY 230 PMPY0028
PMPY 240 PMPY0029
PMPY 250 PMPY0030
PMPY 260 PMPY0031
PMPY 270 PMPY0032
PMPY 280 PMPY0033
PMPY 290 PMPY0034
PMPY 300 PMPY0035
PMPY 310 PMPY0036
PMPY 320 PMPY0037
PMPY 330 PMPY0038

```



```

C          CORRESPONDING EXPONENT OF Z.
C          .....
C          DIMENSION Z(1),X(1),Y(1)
C          IF(IDIMX*IDIMY)10,10,20
C          10 IDIMZ=0
C             GO TO 50
C          20 IDIMZ=IDIMX+IDIMY-1
C          DO 30 I=1,IDIMZ
C             30 Z(I)=0.
C             DO 40 J=1,IDIMX
C                DO 40 J=1,IDIMY
C                   X=I+J-1
C                   40 Z(K)=X(I)*Y(J)+Z(K)
C                   50 RETURN
C                   END
C          520 DIMENSION Z(1),X(1),Y(1)
C          530 DIMENSION Z(1),X(1),Y(1)
C          540 DIMENSION Z(1),X(1),Y(1)
C          550 DIMENSION Z(1),X(1),Y(1)
C          560 DIMENSION Z(1),X(1),Y(1)
C          570 DIMENSION Z(1),X(1),Y(1)
C          580 DIMENSION Z(1),X(1),Y(1)
C          590 DIMENSION Z(1),X(1),Y(1)
C          600 DIMENSION Z(1),X(1),Y(1)
C          610 DIMENSION Z(1),X(1),Y(1)
C          620 DIMENSION Z(1),X(1),Y(1)
C          630 DIMENSION Z(1),X(1),Y(1)
C          640 DIMENSION Z(1),X(1),Y(1)
C          650 DIMENSION Z(1),X(1),Y(1)
C          660 DIMENSION Z(1),X(1),Y(1)
C          670 DIMENSION Z(1),X(1),Y(1)
C          680 DIMENSION Z(1),X(1),Y(1)
C          690 DIMENSION Z(1),X(1),Y(1)
C          700 DIMENSION Z(1),X(1),Y(1)
C          710 DIMENSION Z(1),X(1),Y(1)
C          720 DIMENSION Z(1),X(1),Y(1)
C          730 DIMENSION Z(1),X(1),Y(1)
C          740 DIMENSION Z(1),X(1),Y(1)
C          750 DIMENSION Z(1),X(1),Y(1)
C          760 DIMENSION Z(1),X(1),Y(1)
C          770 DIMENSION Z(1),X(1),Y(1)
C          780 DIMENSION Z(1),X(1),Y(1)
C          790 DIMENSION Z(1),X(1),Y(1)
C          800 DIMENSION Z(1),X(1),Y(1)
C          810 DIMENSION Z(1),X(1),Y(1)
C          820 DIMENSION Z(1),X(1),Y(1)
C          830 DIMENSION Z(1),X(1),Y(1)
C          840 DIMENSION Z(1),X(1),Y(1)
C          850 DIMENSION Z(1),X(1),Y(1)
C          860 DIMENSION Z(1),X(1),Y(1)
C          870 DIMENSION Z(1),X(1),Y(1)
C          880 DIMENSION Z(1),X(1),Y(1)
C          890 DIMENSION Z(1),X(1),Y(1)
C          900 DIMENSION Z(1),X(1),Y(1)
C          910 DIMENSION Z(1),X(1),Y(1)
C          920 DIMENSION Z(1),X(1),Y(1)
C          930 DIMENSION Z(1),X(1),Y(1)
C          940 DIMENSION Z(1),X(1),Y(1)
C          950 DIMENSION Z(1),X(1),Y(1)
C          960 DIMENSION Z(1),X(1),Y(1)
C          970 DIMENSION Z(1),X(1),Y(1)
C          980 DIMENSION Z(1),X(1),Y(1)
C          990 DIMENSION Z(1),X(1),Y(1)

```



```

SUBROUTINE POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
THE ABOVE CARD SHOULD BE PLACED IN PROPER SEQUENCE
BEFORE COMPILING THIS UNDER IBM FORTRAN G.

.....

SUBROUTINE POLRT
PURPOSE
  COMPUTES THE REAL AND COMPLEX ROOTS OF A REAL POLYNOMIAL
USAGE
  CALL POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
DESCRIPTION OF PARAMETERS
  XCOF -VECTOR OF M+1 COEFFICIENTS OF THE POLYNOMIAL
        ORDERED FROM SMALLEST TO LARGEST POWER
  COF  -WORKING VECTOR OF LENGTH M+1
  M    -ORDER OF POLYNOMIAL
  ROOTR-RESULTANT VECTOR OF LENGTH M CONTAINING REAL ROOTS
        OF THE POLYNOMIAL
  ROOTI-RESULTANT VECTOR OF LENGTH M CONTAINING THE
        CORRESPONDING IMAGINARY ROOTS OF THE POLYNOMIAL
  IER  -ERROR CODE WHERE
        IER=0 NO ERROR
        IER=1 M LESS THAN ONE
        IER=2 M GREATER THAN 36
        IER=3 UNABLE TO DETERMINE ROOT WITH 500 INTERATIONS
              ON 5 STARTING VALUES
        IER=4 HIGH ORDER COEFFICIENT IS ZERO
REMARKS
  LIMITED TO 36TH ORDER POLYNOMIAL OR LESS.
  FLOATING POINT OVERFLOW MAY OCCUR FOR HIGH ORDER
  POLYNOMIALS BUT WILL NOT AFFECT THE ACCURACY OF THE RESULTS,

```

```

PLRT 450 PLRT00001
PLRT 450 PLRT00002
PLRT 450 PLRT00003
PLRT 10 PLRT00004
PLRT 20 PLRT00005
PLRT 30 PLRT00006
PLRT 40 PLRT00007
PLRT 50 PLRT00008
PLRT 60 PLRT00009
PLRT 70 PLRT00010
PLRT 80 PLRT00011
PLRT 90 PLRT00012
PLRT 100 PLRT00013
PLRT 110 PLRT00014
PLRT 120 PLRT00015
PLRT 130 PLRT00016
PLRT 140 PLRT00017
PLRT 150 PLRT00018
PLRT 160 PLRT00019
PLRT 170 PLRT00020
PLRT 180 PLRT00021
PLRT 190 PLRT00022
PLRT 200 PLRT00023
PLRT 210 PLRT00024
PLRT 220 PLRT00025
PLRT 230 PLRT00026
PLRT 240 PLRT00027
PLRT 250 PLRT00028
PLRT 260 PLRT00029
PLRT 270 PLRT00030
PLRT 280 PLRT00031
PLRT 290 PLRT00032
PLRT 300 PLRT00033
PLRT 310 PLRT00034
PLRT 320 PLRT00035
PLRT 330 PLRT00036

```



```

SUBROUTINE NEW FUNCTION SUBROUTINE SUBROUTINE
NONE
METHOD
NEWTON-RAPHSON ITERATIVE TECHNIQUE, THE FINAL ITERATIONS
ON EACH ROOT ARE PERFORMED USING THE ORIGINAL POLYNOMIAL
RATHER THAN THE REDUCED POLYNOMIAL TO AVOID ACCUMULATED
ERRORS IN THE REDUCED POLYNOMIAL.
.....
DIMENSION XCOE(1),ROOTR(1),ROOTI(1)
DOUBLE PRECISION XC,VQ,X,Y,XPR,YPR,UX,VY,V,XI2,YI2,SUMSQ,
1 DX,DY,TEMP,ALPHA
.....
IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
STATEMENT WHICH FOLLOWS.
.....
DOUBLE PRECISION XCOE,COE,ROOTR,ROOTI
.....
THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
ROUTINE.
THE DOUBLE PRECISION VERSION MAY BE MODIFIED BY CHANGING THE
CONSTANT IN STATEMENT 78 TO 1.0D-12 AND IN STATEMENT 122 TO
1.0D-10. THIS WILL PROVIDE HIGHER PRECISION RESULTS AT THE
COST OF EXECUTION TIME
.....
IFIT=0
N=M
IFR=0

```

```

PLOT 340 PLOT00037
PLOT 350 PLOT00038
PLOT 360 PLOT00039
PLOT 370 PLOT00040
PLOT 380 PLOT00041
PLOT 390 PLOT00042
PLOT 400 PLOT00043
PLOT 410 PLOT00044
PLOT 420 PLOT00045
PLOT 430 PLOT00046
PLOT 440 PLOT00047
PLOT 450 PLOT00048
PLOT 470 PLOT00049
PLOT 480 PLOT00050
PLOT 490 PLOT00051
PLOT 500 PLOT00052
PLOT 510 PLOT00053
PLOT 520 PLOT00054
PLOT 530 PLOT00055
PLOT 540 PLOT00056
PLOT 550 PLOT00057
PLOT 560 PLOT00058
PLOT 570 PLOT00059
PLOT 580 PLOT00060
PLOT 590 PLOT00061
PLOT 600 PLOT00062
PLOT 610 PLOT00063
PLOT 620 PLOT00064
PLOT 630 PLOT00065
PLOT 640 PLOT00066
PLOT 650 PLOT00067
PLOT 660 PLOT00068
PLOT 670 PLOT00069
PLOT 680 PLOT00070
PLOT 690 PLOT00071
PLOT 700 PLOT00072

```



```

      IF(XCOF(N+1))10,25,10
10  IF(N) 15,15,32
      SET ERROR CODE TO 1
      C
      C
15  IFR=1
20  RETURN
      C
      C
      SET ERROR CODE TO 4
      C
      C
25  IER=4
      GO TO 20
      C
      C
      SET ERROR CODE TO 2
      C
      C
30  IFR=2
      GO TO 20
32  IF(N-36) 35,35,30
35  NX=N
      NXX=N+1
      N2=1
      KJ1 = N+1
      DO 40 I=1,KJ1
      MT=KJ1-I+1
      COF(MT)=XCOF(I)
      C
      C
      SET INITIAL VALUES
      C
      C
45  XC=.00500101
      YC=.01000101
      C
      C
      ZERO INITIAL VALUE COUNTER
      C
      C
      IN=0
      X=XC
      C
      C

```

```

PLRT 710 PLRT0073
PLPT 720 PLPT0074
PLRT 730 PLRT0075
PLPT 740 PLPT0076
PLPT 750 PLPT0077
PLPT 760 PLPT0078
PLRT 770 PLRT0079
PLST 780 PLST0080
PLRT 790 PLRT0081
PLST 800 PLST0082
PLPT 810 PLPT0083
PLPT 820 PLPT0084
PLPT 830 PLPT0085
PLRT 840 PLRT0086
PLPT 850 PLPT0087
PLPT 860 PLPT0088
PLRT 870 PLRT0089
PLRT 880 PLRT0090
PLPT 890 PLPT0091
PLPT 900 PLPT0092
PLPT 910 PLPT0093
PLPT 920 PLPT0094
PLPT 930 PLPT0095
PLPT 940 PLPT0096
PLPT 950 PLPT0097
PLPT 960 PLPT0098
PLRT 970 PLRT0099
PLRT 980 PLRT0100
PLST 990 PLST0101
PLST 000 PLST0102
PLST 010 PLST0103
PLST 020 PLST0104
PLST 030 PLST0105
PLST 040 PLST0106
PLST 050 PLST0107
PLST 060 PLST0108

```


INCREMENT INITIAL VALUES AND COUNTER

$XY = -10, 0^*XY$
 $YD = -10, 0^*X$

SET X AND Y TO CURRENT VALUE

```
X=XQ
Y=YQ
IN=I+J
GO TO 50
IF IT=I
  XPR=X
  YPR=Y
```

EVALUATE POLYNOMIAL AND DERIVATIVES

```

59 ICT=0
60 UX=0.0
61 UY=0.0
62 V =0.0
63 YT=0.0
64 XT=1.0
65 U=COF(N+1)
66 IF(U) 65,130,65
67 DO 70 I=1,N
68 L =N-I+1
69 TEMP=COF(L)
70 XT=X*XT-Y*YT
71 YT=X*YT+Y*XT
72 U=U+TEMP*XT
73 V=V+TEMP*YT
74 FI=I
75 UX=UX+FI*XT*TEMP
76 UY=UY-FI*YT*TEMP
77 XT=YT

```



```

70 YI=YI2
   SUMSQ=UX*UX+UY*UY
   IF(SUMSQ) 75,110,75
75 DX=(V*UY-U*UX)/SUMSQ
   X=X+DX
   DY=-(U*UY+V*UX)/SUMSQ
   Y=Y+DY
78 IF(DABS(DY)+DABS(DX)-1.0D-05) 100,80,80
C
C
C
      STEP ITERATION COUNTER
80 ICT=ICT+1
   IF(ICT-500) 60,85,85
85 IF(IFIT)100,90,100
90 IF(IN-5) 50,95,95
C
C
C
      SET ERROR CODE TO 3
95 IER=3
   GO TO 20
100 DO 105 I=1,NXX
   MT=KJI-L+I
   TEMP=XCOF(MT)
   YCOF(MT)=COF(L)
105 COF(L)=TEMP
   ITEMP=N
   N=NX
   NX=ITEMP
   IF(IFIT) 120,55,120
110 IF(IFIT) 115,50,115
115 X=XD2
   Y=YD2
120 IFIT=0
122 IF(DABS(Y)-1.0D-4*DABS(X)) 135,125,125
125 ALPWA=X+Y
   SUMSQ=X*X+Y*Y

```



```

N=N-2
GO TO 140
130 X=0.0
    NXX=NX-1
    NXX=NXX-1
135 Y=0.0
    SUMSQ=0.0
    ALPHA=X
    N=N-1
140 COF(2)=COF(2)+ALPHA*COF(1)
145 DO 150 I=2,N
150 COF(I+1)=COF(I+1)+ALPHA*COF(I)-SUMSQ*COF(I-1)
155 ROOTI(N2)=Y
    ROOTR(N2)=X
    N2=N2+1
    IF(SUMSQ) 160,165,160
160 Y=-Y
    SUMSQ=0.0
    GO TO 155
165 IF(N) 20,20,45
    END

```

```

PLRT1700 PLRT0181
PLRT1800 PLRT0182
PLRT1810 PLRT0183
PLRT1820 PLRT0184
PLRT1830 PLRT0185
PLRT1840 PLRT0186
PLRT1850 PLRT0187
PLRT1860 PLRT0188
PLRT1870 PLRT0189
PLRT1880 PLRT0190
PLRT1890 PLRT0191
PLRT1900 PLRT0192
PLRT1910 PLRT0193
PLRT1920 PLRT0194
PLRT1930 PLRT0195
PLRT1940 PLRT0196
PLRT1950 PLRT0197
PLRT1960 PLRT0198
PLRT1970 PLRT0199
PLRT1980 PLRT0200
PLRT1990 PLRT0201

```


V. SAMPLE RESULTS

| | SHIP A | SHIP B |
|--------|----------|----------|
| LBF= | 528.50 | 475.00 |
| BEAM= | 76.00 | 68.31 |
| DRAFT= | 29.75 | 26.74 |
| DISPL= | 20900.00 | 15182.00 |
| CB= | 0.6125 | 0.6125 |
| CP= | 0.6246 | 0.6246 |
| CP= | 0.9807 | 0.9807 |

| | SHIP A | SHIP B |
|----------|-------------|--------------|
| DX/DU | -0.8100E 04 | -0.6785E 04. |
| DX/DUDDT | -0.1234E 07 | -0.8960E 06 |
| DY/DV | -0.8123E 05 | -0.6561E 05 |
| DY/DVDDT | -0.2271E 07 | -0.1649E 07 |
| DY/DPCDT | -0.1846E 08 | -0.1340E 08 |
| DY/CPDDT | 0.6677E 07 | 0.4357E 07 |
| DY/DBEL | 0.4898E 06 | 0.3957E 06 |
| DN/DV | -0.9748E 07 | -0.7077E 07 |
| DN/DVDDT | 0.1763E 08 | 0.1150E 08 |
| DN/DPCDT | -0.3246E 10 | -0.2118E 10 |
| DN/CPDDT | -0.3402E 11 | -0.1995E 11 |
| DN/DBEL | -0.1293E 09 | -0.9387E 08 |
| DX/DN | 0.1710E 06 | 0.1242E 06 |
| DY/DN | -0.1937E 05 | -0.1406E 05 |
| DN/DN | 0.5113E 07 | 0.4595E 07 |

AT ALFA= 0.0 STSD= 60.0 AND BETA= 132.2
THE POSITION DEPENDENT DERIVATIVES ARE

| | SHIP A | SHIP B |
|-------|-------------|-------------|
| DY/DA | 0.5399E 00 | 0.4853E 00 |
| DY/DB | -0.2918E 01 | 0.2623E 01 |
| DN/DA | 0.2230E 03 | 0.1801E 03 |
| DN/DB | 0.7403E 03 | -0.5930E 03 |
| DY/DU | 0.1067E 05 | -0.8623E 04 |
| DN/DU | -0.2482E 07 | 0.1802E 07 |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----|-----|-----|
| C.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 |

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----|-----|-----|
| 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 |

SHIP A TIME LAGS ARE 0.0 0.0

SHIP B TIME LAGS ARE 0.0 0.0

THE DETERMINANT IN ASCENDING D
POWERS IS

| | |
|------------|--|
| 0.0 | |
| 0.0 | |
| C.1255E 32 | |
| C.5102E 34 | |
| C.9221E 36 | |
| C.1460E 39 | |
| 0.2186E 41 | |
| 0.1949E 43 | |
| C.7658E 44 | |
| 0.3479E 45 | |
| C.2802E 46 | |

PCURT TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

| |
|-------------|
| 0.0 |
| 0.0 |
| -0.1250E-01 |
| -0.8846E-02 |
| -0.6807E-02 |
| -0.1280E 00 |
| -0.7573E-02 |
| 0.1730E-02 |
| 0.1730E-02 |
| -0.1424E 00 |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----------|----------|-----------|
| 1.000000 | 4.000000 | -0.010000 |
| -0.100000 | 0.010000 | 0.200000 |

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|----------|-----------|-----------|
| 1.000000 | 4.000000 | 0.010000 |
| 0.100000 | -0.010000 | -0.200000 |

| | | |
|----------------------|-----|-----|
| SHIP A TIME LAGS ARE | 0.0 | 0.0 |
| SHIP B TIME LAGS ARE | 0.0 | 0.0 |

THE DETERMINANT IN ASCENDING D
POWERS IS

| | |
|---------|----|
| 0.1520E | 34 |
| 0.3168E | 36 |
| 0.1883E | 38 |
| 0.6835E | 39 |
| 0.1623E | 41 |
| 0.2645E | 42 |
| 0.2997E | 43 |
| 0.2399E | 44 |
| 0.1652E | 45 |
| 0.9746E | 45 |
| 0.2802E | 46 |

FCLRT TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

| |
|-------------|
| -0.6400E-01 |
| -0.7190E-02 |
| -0.1149E 00 |
| -0.1745E-01 |
| -0.1745E-01 |
| -0.3131E-01 |
| -0.3131E-01 |
| -0.1291E 00 |
| 0.3243E-01 |
| 0.3243E-01 |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----------|----------|-----------|
| 1.000000 | 4.000000 | -0.010000 |
| -0.100000 | 0.010000 | 0.200000 |

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|----------|-----------|-----------|
| 1.000000 | 4.000000 | 0.010000 |
| 0.100000 | -0.010000 | -0.200000 |

| | | |
|----------------------|------|------|
| SHIP A TIME LAGS ARE | 2.00 | 4.00 |
|----------------------|------|------|

| | | |
|----------------------|------|------|
| SHIP B TIME LAGS ARE | 2.00 | 4.00 |
|----------------------|------|------|

THE DETERMINANT IN ASCENDING D
POWERS IS

| | |
|---------|----|
| 0.1520E | 34 |
| 0.3046E | 36 |
| 0.1636E | 38 |
| 0.5489E | 39 |
| 0.1209E | 41 |
| 0.1821E | 42 |
| 0.1936E | 43 |
| 0.1497E | 44 |
| 0.1084E | 45 |
| 0.7140E | 45 |
| 0.2221E | 46 |

POLRI TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

| |
|-------------|
| -0.6552E-01 |
| -0.7190E-02 |
| -0.3311E-01 |
| -0.3311E-01 |
| -0.1124E 00 |
| -0.1631E-01 |
| -0.1631E-01 |
| -0.1261E 00 |
| 0.4432E-01 |
| 0.4432E-01 |

AT ALFA= 0.0 STSD= 90.0 AND BETA= 162.2

THE POSITION DEPENDENT DERIVATIVES ARE

| | SHIP A | SHIP B |
|-------|-------------|-------------|
| DY/DA | 0.2733E 00 | 0.2456E 00 |
| EY/EB | -0.1918E 01 | 0.1724E 01 |
| DN/DA | 0.1066E 03 | 0.8611E 02 |
| EN/DB | 0.4759E 03 | -0.3844E 03 |
| EY/DU | 0.7685E 04 | -0.6208E 04 |
| DN/DU | -0.1715E 07 | 0.1245E 07 |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----|-----|-----|
| C.O | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 |

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----|-----|-----|
| 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 |

SHIP A TIME LAGS ARE 0.0 C.O

SHIP B TIME LAGS ARE 0.0 0.0

THE DETERMINANT IN ASCENDING C
POWERS IS

| |
|------------|
| 0.0 |
| C.O |
| C.8124E 31 |
| 0.3302E 34 |
| C.6707E 36 |
| C.1329E 39 |
| C.2171E 41 |
| 0.1949E 43 |
| C.7658E 44 |
| 0.8479E 45 |
| C.2802E 46 |

PCLRT TEST VALUE IER= 0

CHAR. EGN. REAL ROOTS ARE

| |
|-------------|
| 0.0 |
| 0.0 |
| -0.1165E-01 |
| -0.8832E-02 |
| -0.6808E-02 |
| -0.1280E 00 |
| -0.7571E-02 |
| 0.1296E-02 |
| 0.1296E-02 |
| -0.1424E 00 |

| | | |
|----------------------|-----|-----|
| SHIP A TIME LAGS ARE | 0.0 | 0.0 |
| SHIP B TIME LAGS ARE | 0.0 | 0.0 |

THE DETERMINANT IN ASCENDING D
POWERS IS

| | |
|---------|----|
| 0.1519E | 34 |
| 0.3167E | 36 |
| 0.1883E | 38 |
| 0.6833E | 39 |
| 0.1622E | 41 |
| 0.2645E | 42 |
| 0.2996E | 43 |
| 0.2399E | 44 |
| 0.1652E | 45 |
| 0.9746E | 45 |
| 0.2802E | 46 |

FCLRT TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

| |
|-------------|
| -0.6399E-01 |
| -0.7190E-02 |
| -0.1149E 00 |
| -0.1745E-01 |
| -0.1745E-01 |
| -0.3131E-01 |
| -0.3131E-01 |
| -0.1291E 00 |
| 0.3243E-01 |
| 0.3243E-01 |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----------|----------|-----------|
| 1.000000 | 4.000000 | -0.010000 |
| -0.100000 | 0.010000 | 0.200000 |

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|----------|-----------|-----------|
| 1.000000 | 4.000000 | 0.010000 |
| 0.100000 | -0.010000 | -0.200000 |

SHIP A TIME LAGS ARE 2.00 4.00

SHIP B TIME LAGS ARE 2.00 4.00

THE DETERMINANT IN ASCENDING D
POWERS IS

| | |
|---------|----|
| 0.1519E | 34 |
| 0.3046E | 36 |
| 0.1636E | 38 |
| 0.5488E | 39 |
| 0.1208E | 41 |
| 0.1821E | 42 |
| 0.1936E | 43 |
| 0.1497E | 44 |
| 0.1084E | 45 |
| 0.7140E | 45 |
| 0.2221E | 46 |

PCURT TEST VALUE IER= 0

CHAR. EGN. REAL ROOTS ARE

| |
|-------------|
| -0.6552E-01 |
| -0.7190E-02 |
| -0.3311E-01 |
| -0.3311E-01 |
| -0.1124E 00 |
| -0.1631E-01 |
| -0.1631E-01 |
| -0.1261E 00 |
| 0.4431E-01 |
| 0.4431E-01 |

AT ALFA= 200.0 SISO= 70.0 AND BETA= 142.2

THE POSITION DEPENDENT DERIVATIVES ARE

| | SHIP A | SHIP B |
|-------|-------------|-------------|
| EY/CA | -0.6247E 00 | 0.5186E 00 |
| DY/CB | -0.6854E-01 | -0.1170E 01 |
| EN/CA | 0.3878E 02 | -0.2561E 02 |
| EN/CB | -0.2293E 03 | -0.4629E 02 |
| EY/DA | 0.2351E 04 | 0.1942E 04 |
| EN/DA | 0.8242E 06 | 0.2333E 06 |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----|-----|-----|
| 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 |

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----|-----|-----|
| 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 |

SHIP A TIME LAGS ARE 0.0 0.0

SHIP B TIME LAGS ARE 0.0 0.0

THE DETERMINANT IN ASCENDING C
POWERS IS

| | |
|-------------|--|
| 0.0 | |
| 0.0 | |
| -0.1766E 31 | |
| -0.7038E 33 | |
| 0.1157E 36 | |
| 0.1044E 39 | |
| 0.2141E 41 | |
| 0.1949E 43 | |
| 0.7659E 44 | |
| 0.8479E 45 | |
| 0.2802E 46 | |

PCLRT TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

| | |
|-------------|--|
| 0.0 | |
| 0.0 | |
| -0.9162E-02 | |
| -0.7577E-02 | |
| -0.6806E-02 | |
| -0.1279E 00 | |
| -0.7340E-02 | |
| 0.2534E-02 | |
| -0.1424E 00 | |
| -0.3938E-02 | |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----------|----------|-----------|
| 1.000000 | 4.000000 | -0.010000 |
| -0.100000 | 0.010000 | 0.200000 |

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|----------|-----------|-----------|
| 1.000000 | 4.000000 | 0.010000 |
| 0.100000 | -0.010000 | -0.200000 |

| | | |
|----------------------|-----|-----|
| SHIP A TIME LAGS ARE | 0.0 | 0.0 |
| SHIP B TIME LAGS ARE | 0.0 | 0.0 |

THE DETERMINANT IN ASCENDING D
POWERS IS

| | |
|---------|----|
| 0.1519E | 34 |
| 0.3166E | 36 |
| 0.1882E | 38 |
| 0.6031E | 39 |
| 0.1622E | 41 |
| 0.2644E | 42 |
| 0.2996E | 43 |
| 0.2399E | 44 |
| 0.1652E | 45 |
| 0.9746E | 45 |
| 0.2802E | 46 |

PCLRT TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

| |
|-------------|
| -0.6397E-01 |
| -0.7190E-02 |
| -0.1149E 00 |
| -0.1745E-01 |
| -0.1745E-01 |
| -0.3131E-01 |
| -0.3131E-01 |
| -0.1291E 00 |
| 0.3241E-01 |
| 0.3241E-01 |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----------|----------|-----------|
| 1.000000 | 4.000000 | -0.010000 |
| -0.100000 | 0.010000 | 0.200000 |

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|----------|-----------|-----------|
| 1.000000 | 4.000000 | 0.010000 |
| 0.100000 | -0.010000 | -0.200000 |

| | | |
|----------------------|------|------|
| SHIP A TIME LAGS ARE | 2.00 | 4.00 |
|----------------------|------|------|

| | | |
|----------------------|------|------|
| SHIP B TIME LAGS ARE | 2.00 | 4.00 |
|----------------------|------|------|

THE DETERMINANT IN ASCENDING D
POWERS IS

| | |
|---------|----|
| 0.1519E | 34 |
| 0.3045E | 36 |
| 0.1635E | 38 |
| 0.5486E | 39 |
| 0.1208E | 41 |
| 0.1820E | 42 |
| 0.1936E | 43 |
| 0.1497E | 44 |
| 0.1085E | 45 |
| 0.7140E | 45 |
| 0.2221E | 46 |

POLRT TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

| |
|-------------|
| -0.6550E-01 |
| -0.7190E-02 |
| -0.3311E-01 |
| -0.3311E-01 |
| -0.1124E 00 |
| -0.1631E-01 |
| -0.1631E-01 |
| -0.1261E 00 |
| 0.4420E-01 |
| 0.4430E-01 |

AT ALFA= -400.0 STD= 30.0 AND BETA= 152.2

THE POSITION DEPENDENT DERIVATIVES ARE

| | SHIP A | SHIP B |
|-------|-------------|-------------|
| DY/EA | 0.1457E 00 | -0.2734E 00 |
| DY/EB | 0.1492E 01 | -0.3327E 00 |
| DN/EA | -0.1208E 03 | -0.4958E 02 |
| DN/EB | -0.2523E 03 | 0.1403E 03 |
| DY/ED | -0.3333E 04 | -0.2426E 02 |
| DN/ED | 0.6393E 06 | -0.4488E 06 |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----|-----|-----|
| 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 |

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6

| | | |
|-----|-----|-----|
| 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 |

| | | |
|----------------------|-----|-----|
| SHIP A TIME LAGS ARE | 0.0 | 0.0 |
| SHIP B TIME LAGS ARE | 0.0 | 0.0 |

THE DETERMINANT IN ASCENDING D
POWERS IS

| |
|-------------|
| 0.0 |
| 0.0 |
| -0.3609E 31 |
| -0.1458E 34 |
| 0.8493E 34 |
| 0.9850E 38 |
| 0.2132E 41 |
| 0.1948E 43 |
| 0.7659E 44 |
| 0.8479E 45 |
| 0.2802E 46 |

PCLRT TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

| |
|-------------|
| 0.0 |
| 0.0 |
| -0.9157E-02 |
| -0.7574E-02 |
| -0.6129E-02 |
| -0.6129E-02 |
| -0.1279E 00 |
| 0.3502E-02 |
| -0.1424E 00 |
| -0.6806E-02 |

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
 1.000000 4.000000 -0.010000
 -0.100000 0.010000 0.200000

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
 1.000000 4.000000 0.010000
 0.100000 -0.010000 -0.200000

SHIP A TIME LAGS ARE 0.0 0.0
 SHIP B TIME LAGS ARE 0.0 0.0

THE DETERMINANT IN ASCENDING D
 POWERS IS

0.1519E 34
 0.3166E 36
 0.1982E 38
 0.6831E 39
 0.1622E 41
 0.2644E 42
 0.2996E 43
 0.2399E 44
 0.1652E 45
 0.9746E 45
 0.2902E 46

PCLRT TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

-0.6397E-01
 -0.7190E-02
 -0.1149E 00
 -0.1745E-01
 -0.1745E-01
 -0.3131E-01
 -0.3131E-01
 -0.1291E 00
 0.3241E-01
 0.3241E-01

SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
 1.000000 4.000000 -0.010000
 -0.100000 0.010000 0.200000

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
 1.000000 4.000000 0.010000
 0.100000 -0.010000 -0.200000

SHIP A TIME LAGS ARE 2.00 4.00
 SHIP B TIME LAGS ARE 2.00 4.00

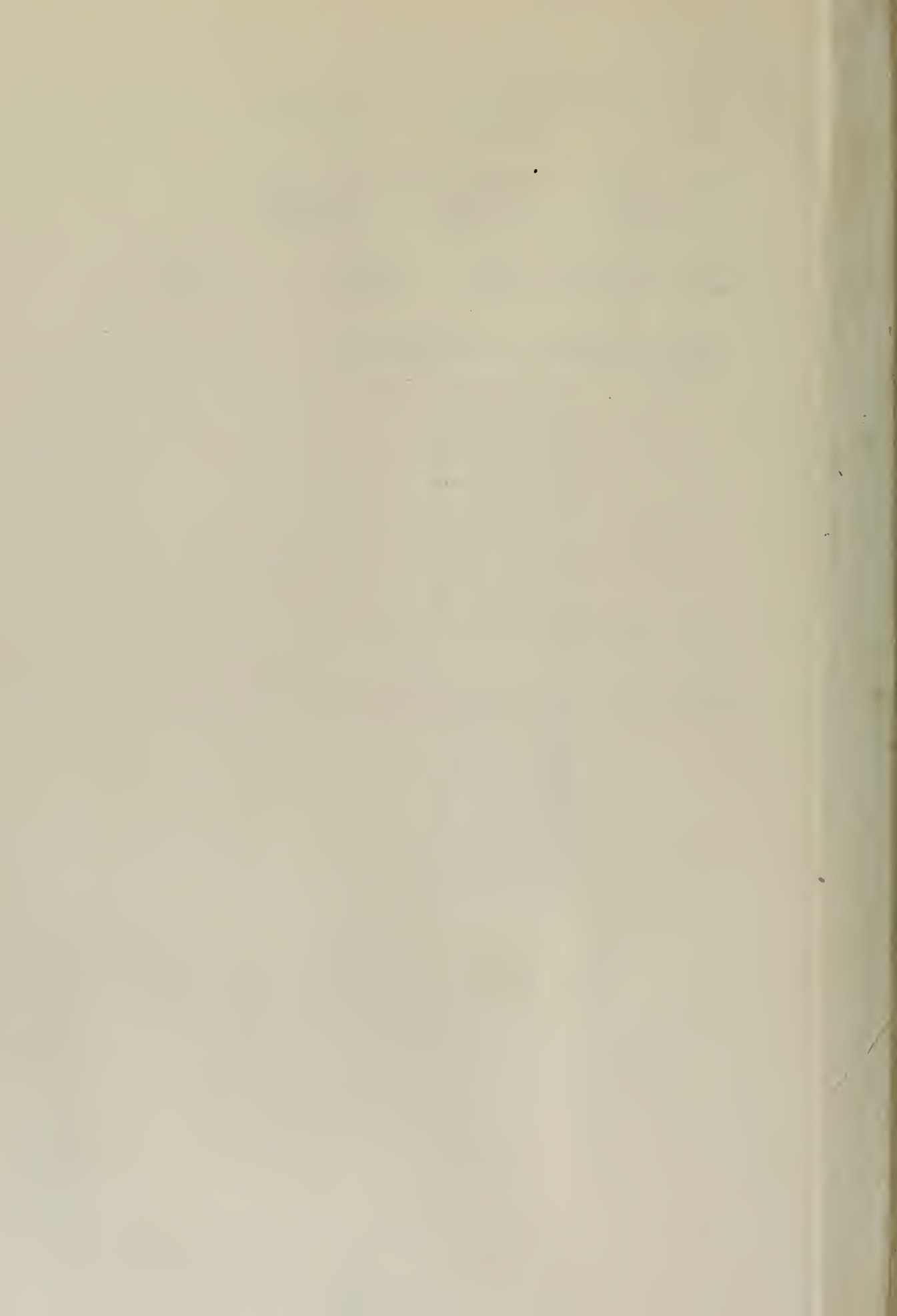
THE DETERMINANT IN ASCENDING D
 POWERS IS

0.1519E 34
 0.3044E 36
 0.1635E 38
 0.5486E 39
 0.1208E 41
 0.1820E 42
 0.1936E 43
 0.1497E 44
 0.1085E 45
 0.7140E 45
 0.2221E 46

FCLRT TEST VALUE IER= 0

CHAR. EQN. REAL ROOTS ARE

-0.6550E-01
 -0.7190E-02
 -0.3311E-01
 -0.3311E-01
 -0.1124E 00
 -0.1631E-01
 -0.1631E-01
 -0.1261E 00
 0.4429E-01
 0.4429E-01



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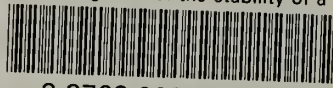
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An investigation of
the stability of a
system of two ships
employing automatic
control while on paral-
lel courses in close
proximity.

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